

Statistical Anisotropy from Vector Curvaton in D-brane Inflation

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ABSTRACT: We discuss an explicit realisation of the vector curvaton paradigm in D-brane models of inflation in Type IIB string theory. The vector curvaton is identified with the U(1) gauge field that lives on the world volume of a D3-brane, which may be stationary or undergoing general motion in the internal space. The dilaton is considered as a spectator field which modulates the evolution of the vector field. We demonstrate that the vector curvaton is able to generate measurable statistical anisotropy in the spectrum and bispectrum of the curvature perturbation as long as the dilaton evolves as $e^{-\phi} \propto a^2$ where $a(t)$ is the scale factor.

KEYWORDS: Vector Curvaton, D-brane Inflation, Statistical Anisotropy, non-Gaussianity.

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1. Introduction

1.1 Cosmic Inflation and the Vector Curvaton paradigm

Ever-improving galaxy surveys have revealed to us that structure on the largest cosmological scales has an intricate net-like configuration, aptly named the Cosmic Web. According to the current model of structure formation, the gravitationally entangled strands that we see today have evolved from small density perturbations that left their imprint on the oldest source that is directly observed: the Cosmic Microwave Background (CMB). To understand how these “seeds of structure” came to be is an important challenge for cosmology.

The CMB also provides us with the strongest observational evidence for the homogeneity and isotropy of the universe on the largest scales, as it is uniform to about one part in ten thousand [1]. We can then ask the question: What sort of processes or events in the earliest cosmological moments were able to make the universe so uniform, and yet still implant the tiny deviations from uniformity that could subsequently grow into the rich galaxy systems that we observe today?

The most successful framework for answering this question is the idea that the universe underwent a period of rapid, exponential or quasi-exponential expansion early on in its history, which on the one hand drove all classical perturbations to zero, but on the other hand was able to amplify fluctuations of the vacuum to set up the initial conditions for structure growth. This framework, dubbed cosmic inflation, also provides compelling insight into other seemingly unrelated problems in cosmology, such as the problem of the flatness of space and the topological defects problem.

The amplification of vacuum fluctuations is of particular interest because, while the other problems can be overcome or at least ameliorated by *any* models that are able to generate sufficiently long-lasting inflation, producing a spectrum of amplified fluctuations with the correct properties (as dictated by observations), necessitates the use of a more stringent approach. Such properties are manifest in the *curvature perturbation* ζ which is generated by the dominant fluctuations, and typically include a high degree of scale invariance, of Gaussianity and of statistical isotropy and homogeneity. As a first test, all viable models of inflation must be able to reproduce these features. However, precision measurements of the CMB have revealed the presence of finer-grained deviations from these basic properties which appear robust against foreground removal. For example, the low multipoles of the CMB appear to be in alignment, which hints at the existence of a preferred direction on the microwave sky and therefore may constitute a violation of statistical isotropy [2]. The capacity to provide a concrete explanation for the appearance of these deviations forms the finest sieve for models of inflation.

Many inflation models assume that the energy density for inflating the universe and the vacuum fluctuations for seeding the galaxies must be provided by one and the same field, making it difficult to find a candidate field with all the right traits.

There is however no *a priori* reason to assume that this should be the case, given that the two jobs are rather independent. Indeed, any field that is around at the time of the expansion may end up having its vacuum fluctuations amplified, so long as it is sufficiently light and conformally non-invariant, regardless of which field is driving the expansion. If such a field is subsequently able to affect the expansion of the Universe then its own perturbation spectrum may be imprinted in the form of the curvature perturbation. One way for a field to do so is by dominating the energy density of the Universe after inflation, an idea that has been fruitfully expounded upon in the curvaton paradigm [3]. Such a field, which has nothing to do with expanding the Universe but instead does the job of generating the dominant contribution to ζ , is then referred to as the *curvaton* field.

Scalar fields, in addition to their wide use as inflatons, have also been popularly studied as curvatons, because if such fields are able to dominate the energy density of the Universe to imprint their spectrum, they will do so in an inherently isotropic way. A vector field on the other hand features an opposite sign for the pressure along its longitudinal direction relative to that along its transverse directions. Hence, if such a field (homogenised by inflation) were able to dominate the radiation background so as to imprint its own spectrum onto the curvature perturbation, it is likely to induce substantial anisotropic stress leading to excessive large-scale anisotropy when its pressure is non-vanishing, which is not observed. However, it has been shown that an oscillating vector field behaves like pressureless isotropic matter, therefore, as long as the vector field begins to oscillate before its density parameter becomes significant, it may indeed dominate the radiation background in an observationally consistent way [4]. Therefore, a vector field can, in principle, play the role of the curvaton.

The process by which the fluctuations of the vacuum are amplified in an expanding space is known as gravitational particle production, for the appearance of real particles in such a background is interpreted as the creation of particles by the changing gravitational field [5]. A second problem which arises when using a vector field as a curvaton is related to this particle production process. A field must be sufficiently light to undergo gravitational particle production. However, a massless vector field is conformally invariant, therefore it does not respond to the expanding background and its vacuum fluctuations do not become amplified. In Ref. [4] the consequences of including a small non-zero vector mass were examined, and it was found that a vector field may indeed undergo particle production and obtain a scale invariant superhorizon spectrum of perturbations if the mass satisfies $m^2 \approx -2H_*^2$ during inflation, where H_* gives the inflationary Hubble scale. This work pioneered the *vector curvaton* scenario, demonstrating for the first time that it is possible for the curvature perturbation to be affected or even generated by a vector field. The idea was implemented by coupling the vector field non-minimally to gravity through

a $\frac{1}{6}RA^2$ term in Refs. [6, 7].

The scenario was further developed in Ref. [8], where the supergravity motivated case of a vector field with a varying gauge kinetic function is explored as an alternative to demanding the afore-mentioned mass-squared condition. There it was shown that the transverse components of such a vector field can indeed produce a scale invariant spectrum of superhorizon perturbations, but the longitudinal component was not considered. In Ref. [9], the significance of including the longitudinal component was brought to light: it then becomes possible to generate a statistically anisotropic contribution to the curvature perturbation, at once providing an explanation for the appearance of such a feature within a concrete paradigm. (For a recent review see Ref. [10].)¹ In the case that the statistical anisotropy in the spectrum generated by the vector field is large, the dominant contribution to ζ must come from an isotropic source, such as the scalar inflaton. The vector curvaton then affects the curvature perturbation such that it acquires a measureable degree of statistical anisotropy.

1.2 The D-brane Vector Curvaton

The idea that more than one field might have a role to play in the evolution of the early universe is highly motivated by string theory, which generically contains many fields, even at energies far below both the string and compactification scales. In standard inflation scenarios, the focus is often placed purely upon the behaviour of the candidate inflaton, while the other fields present in the set-up are considered to be either negligible or stabilised at the minima of their respective potentials. In the light of the curvaton scenario, it is interesting to consider how these fields might impact the evolution of the universe if they themselves are allowed to evolve. In particular, in open string D-brane models of inflation, the inflaton is typically identified with the position coordinate of a Dp -brane moving in a warped throat [12]. Such a brane features a world volume two form field F_{MN} , associated with the open strings that end on it. The components of such a field which correspond to Wilson lines have been studied as potential inflaton candidates, both in the unwarped and warped cases in Refs. [13, 14]. Thus it is natural to investigate the role of the other components, in particular the 4D components, of such a field during the cosmological evolution. This indeed suggests the possibility to embed the vector curvaton paradigm in D-brane models of inflation.

In the context of these models, the prototypical scenario of slow-roll inflation has been described by the motion of a D3-brane along a particularly flat section of its potential in the throat, yielding at once a firm embedding for this scenario within a fundamental theory. Interestingly, further investigation of this picture within string theory led to an unexpected result: Inflation can take place even when the potential

¹In these works the vector field is assumed to be Abelian. For non-Abelian vector fields and their potential contribution to ζ see Ref. [11].

is very steep, leading to a completely new type of inflation known as Dirac-Born-Infeld (DBI) inflation [15]. In DBI inflation, one no longer assumes that the kinetic term for the position field has its canonical form, but rather keeps the non-standard form just as it appears in the DBI action which describes the motion of the brane. This amounts to allowing the brane to move relativistically, bounded by the natural speed limit in the bulk, and the combination of this speed limit with strong warping in the throat allows for inflationary trajectories.

In the same way, the $U(1)$ vector field whose kinetic term appears in the DBI action is often considered to be of canonical form. However, in analogy with DBI inflation, keeping the general form may lead to new insights in cosmology if this vector field is considered to play a role in the cosmological evolution. While a single vector field is unsuitable for the role of the inflaton, the vector curvaton paradigm has demonstrated that it can indeed affect or even generate the curvature perturbation. In this work we therefore explore the cosmological implications of the canonical as well as the non-canonical D-brane vector field when treated as a curvaton field, working within the picture of $D3\overline{D3}$ brane inflation in a warped Type IIB flux compactification [12].

Another novel feature that we explore is the role of the dilaton during the cosmological evolution, not as an inflaton but rather as a “modulon”: a degree of freedom that varies during inflation and modulates the mass and gauge kinetic function of the vector field [9]. We show that allowing the dilaton to modulate the vector field in this way can lead naturally to the appearance of measurable statistical anisotropy in the spectrum and bispectrum of the curvature perturbation, assuming that the dilaton evolves as $e^\phi \propto a^{-2}$ where $a(t)$ is the scale factor. Our incorporation of the possible dynamics of the dilaton into the cosmological evolution is motivated by the recent study in Ref. [16], in which it is shown that, while the $D3\overline{D3}$ potential receives various corrections from objects and fields in the bulk, small and large gradients of the dilaton do not seem to affect the potential.

In what follows we begin by making the simple assumption that the D-brane vector field, which is here considered a curvaton, is living on the world volume of a stationary D3-brane. Taking the dilaton to be stabilised at first, we calculate the equations of motion for the perturbations of this D-brane vector field in its full non-canonical form, *i.e.* just as it appears in the DBI action. We show that many new terms arise in the equations of motion for the perturbations of the vector field, many of them exhibiting clear anisotropies as well as couplings to the vector background. In general these terms are subdominant when the vector background freezes out during inflation. We then take the canonical limit, but add the new feature of an evolving dilaton. Equations of this form (those which describe the perturbations of a modulated vector field) have been solved explicitly in Ref. [9] where it was first shown that statistical anisotropy can arise naturally from such a scenario, so in the present work we propose a concrete realisation of this scenario in string theory.

We then consider the host brane for our vector curvaton to be a moving brane, in particular the brane whose varying position coordinate is driving inflation. We show that our results are robust against such a change, whether the brane is slowly rolling or moving relativistically. We briefly comment on the generalisation to a multifield scenario.

Our model is therefore the first string realisation of the vector curvaton paradigm. In particular, we provide a concrete model within string theory for the scenario studied in Ref. [9], in which the dominant contribution to the curvature perturbation is given by the inflaton field, and the vector field can contribute measurable statistical anisotropy. Throughout our paper we use natural units for which $c = \hbar = k_B = 1$ and Newton's gravitational constant is $8\pi G = M_P^{-2}$, with $M_P = 2.4 \times 10^{18}$ GeV being the reduced Planck mass. Sometimes we revert to geometric units where $M_P = 1$.

2. The General Set-up

In this section we discuss the set up which will be the basis for our investigation of the D-brane vector curvaton with non-standard kinetic terms.

We consider a warped flux compactification in type IIB theory [17, 18]. Thus, the ten dimensional metric takes the following form (in the Einstein frame)

$$G_{MN}dx^M dx^N = h^{-1/2}(y^A) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^A) g_{AB} dy^A dy^B. \quad (2.1)$$

Here h is the warp factor which depends on the compact coordinates y^A , and g_{AB} is the internal metric which may also depend on the compact coordinates. This geometry is the result of having all types of fluxes present in the theory turned on: RR forms $F_{n+1} = dC_n$ for $n = 0, 2, 4$ and their duals $n = 6, 8$ (remember also that F_5 is self dual), as well as NSNS flux $H_3 = dB_2$. These fluxes have only compact internal components, therefore their duals have legs in all four infinite dimensions plus the relevant components in the internal dimensions.

We now consider a probe D p -brane (or anti-brane) embedded in this background. This has four of its dimensions lying parallel to the four infinite dimensions, and $(p - 3)$ spatial dimensions wrapped along an internal $(p - 3)$ -cycle. The dynamics of such a brane is described by the sum of the Dirac-Born-Infeld (DBI) and Wess-Zumino (WZ) actions. The DBI part is given, in the Einstein frame, by²

$$S_{\text{DBI}} = -\mu_p \int d^{p+1}\xi e^{\frac{(p-3)}{4}\phi} \sqrt{-\det(\gamma_{ab} + e^{-\frac{\phi}{2}} \mathcal{F}_{ab})}, \quad (2.2)$$

²In the string frame, the DBI action is given by

$$S_{\text{DBI}} = -\mu_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det(\gamma_{ab} + \mathcal{F}_{ab})}.$$

In D dimensions, the Einstein and string frames are related by $G_{MN}^E = e^{-\frac{4}{(D-2)}\phi} G_{MN}^s$.

where the tension of a Dp -brane in the Einstein frame is³

$$T_p = \mu_p e^{\frac{(p-3)}{4}\phi} \quad \text{with} \quad \mu_p = (2\pi)^{-p} (\alpha')^{-(p+1)/2},$$

where ϕ is the dilaton. Furthermore, $\mathcal{F}_{ab} = \mathcal{B}_{ab} + 2\pi\alpha' F_{ab}$, with \mathcal{B}_2 the pullback of the NSNS background 2-form field on the brane, F_2 is the world volume gauge field we are interested in, and $\gamma_{ab} = G_{MN} \partial_a x^M \partial_b x^N$ is the pullback of the ten-dimensional metric on the brane in the Einstein frame. Finally, $\alpha' = \ell_s^2$ is the string scale and ξ^a are the brane world-volume coordinates. The indices $M, N = 0, 1, \dots, 9$; $a, b = 0, 1, \dots, p$; $\mu, \nu = 0, 1, \dots, 3$ and $A, B = 4, \dots, 9$ label the coordinates in 10D spacetime, on the worldvolume, in 4D spacetime including the three extended dimensions, and in the remaining six internal dimensions respectively. The action in Eq. (2.2) is reliable for arbitrary values of the gradients $\partial_a x^M$, so long as these are themselves slowly varying in spacetime; that is, for small accelerations compared to the string scale (equivalently, for small extrinsic curvatures of the brane worldvolume). In addition, the string coupling should be small in order for the perturbative expansion to hold, *i.e.* $g_s \ll 1$, where $g_s = e^{\phi_0}$.

The Wess-Zumino action describing the coupling of the Dp -brane to the RR form fields is given by

$$S_{\text{WZ}} = q \mu_p \int_{\mathcal{W}_{p+1}} \sum_n \mathcal{C}_n \wedge e^{\mathcal{F}}, \quad (2.3)$$

where \mathcal{C}_n are the pullbacks of the background RR C_n forms present, $\mathcal{F} = \mathcal{B} + 2\pi\alpha' F$ as before, and the wedge product singles out the relevant terms in the exponential. Furthermore \mathcal{W}_{p+1} is the world volume of the brane and $q = 1$ for a probe Dp -brane, while $q = -1$ for a probe Dp -antibrane.

We now discuss these two actions in detail for our case of interest, a D3-brane.

2.1 The Dirac-Born-Infeld Action

Let us consider a probe D3-brane positioned such that its three axes are aligned with the axes of the three extended spatial dimensions. We consider the brane to be moving in the internal space and so the internal coordinates become functions of the world volume coordinates, $y^A = y^A(\xi^\mu)$. In typical single-field inflation scenarios with D3-branes/ $\overline{\text{D3}}$ -antibranes, the inflaton field is identified with the position coordinate of the D3-brane moving radially in the potential of the antibrane. Taking the pullback of the NSNS 2-form field \mathcal{B}_2 to vanish, the DBI action for a D3-brane is given by

$$S_{\text{DBI}} = -T_3 \int d^4x \sqrt{-\det(\gamma_{\mu\nu} + e^{-\phi/2} \mathcal{F}_{\mu\nu})}, \quad (2.4)$$

where

$$\gamma_{\mu\nu} = h(r)^{-1/2} g_{\mu\nu} + h(r)^{1/2} \partial_\mu y^A \partial_\nu y^B g_{AB}, \quad \mathcal{F}_{\mu\nu} = 2\pi\alpha' F_{\mu\nu},$$

³Notice that for a D3-brane, $T_p = \mu_p$ in the Einstein frame.

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We expand the determinant as follows:

$$-\det[h^{-1/2}g_{\mu\nu} + h^{1/2}\partial_\mu y^A \partial_\nu y_A + 2\pi\alpha' e^{-\phi/2} F_{\mu\nu}] = -h^{-2}\det[g_{\mu\beta}]\det[\delta^\beta_\nu + h\partial^\beta y^A \partial_\nu y_A + lF^\beta_\nu]$$

$$l = h^{1/2}2\pi\alpha' e^{-\phi/2}.$$

The DBI action in Eq. (2.4) then becomes

$$S_{\text{DBI}} = -T_3 \int d^4x h^{-1} \sqrt{-g} \sqrt{\det(\delta^\beta_\nu + h\partial^\beta y^A \partial_\nu y_A + lF^\beta_\nu)}. \quad (2.5)$$

We now proceed to calculate the determinant:

$$\begin{aligned} \det[\delta^\beta_\nu + h\partial^\beta y^A \partial_\nu y_A + lF^\beta_\nu] &= 1 + \frac{l^2}{2} F_{\alpha\beta} F^{\alpha\beta} - \frac{l^4}{4} F_{\alpha\beta} F^{\beta\gamma} F_{\gamma\delta} F^{\delta\alpha} + \frac{l^4}{8} F_{\alpha\beta} F^{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta} \\ &+ h y_A^\alpha y_\alpha^A + h^2 y_A^{[\alpha} y_\alpha^A y_B^\beta y_\beta^B + h^3 y_A^{[\alpha} y_\alpha^A y_B^\beta y_\beta^B y_\gamma^\gamma y_\gamma^C + h^4 y_A^{[\alpha} y_\alpha^A y_B^\beta y_\beta^B y_\gamma^\gamma y_\gamma^C y_\delta^\delta y_\delta^D \\ &+ 3hl^2 y_A^{[\alpha} y_\alpha^A F_{\gamma\beta}^\beta F_{\gamma}^\gamma]_\beta + 3h^2 l y_A^{[\alpha} y_\alpha^A y_B^\beta y_\beta^B F_{\gamma}^\gamma]_\beta + 4h^3 l y_A^{[\alpha} y_\alpha^A y_B^\beta y_\beta^B y_\gamma^\gamma y_\gamma^C F_{\delta}^\delta]_\gamma + 4hl^3 y_A^{[\alpha} y_\alpha^A F_{\gamma\delta}^\beta F_{\gamma}^\gamma F_{\delta}^\delta]_\beta \\ &+ 6h^2 l^2 y_A^{[\alpha} y_\alpha^A y_B^\beta y_\beta^B F_{\delta}^\delta F_{\gamma}^\gamma]_\gamma, \end{aligned} \quad (2.6)$$

where $y_A^\alpha \equiv \partial^\alpha y_A$ and the antisymmetrisation takes place over the Greek indices only.

2.2 The Wess Zumino Action

The general Wess Zumino (WZ) action for a D3-brane is given by (see Eq. (2.3))

$$S_{\text{WZ}} = q\mu_3 \int_{\mathcal{W}_4} \left(\mathcal{C}_4 + \mathcal{C}_2 \wedge (2\pi\alpha') F_2 + \frac{\mathcal{C}_0 (2\pi\alpha')^2}{2} F_2 \wedge F_2 \right), \quad (2.7)$$

where we recall that the \mathcal{C}_n are the pullbacks of the background C_n forms present in the flux background, \mathcal{W}_4 is the world volume, and q gives the charge of the brane ($q = +1$ for a brane and $q = -1$ for an antibrane).

The first term in Eq. (2.7) simply gives the charge of the D3-brane. In a flux compactification, the four form is given by $\mathcal{C}_4 = \sqrt{-g} h^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$ and thus this term is essentially given by the warp factor. The last term in Eq. (2.7) is the coupling of the axion field C_0 to the vector field we are interested in. In the present case where the C_0 axion has been stabilised, this is just a topological term. Finally, the second term which couples the vector field to the two-form, is a non-trivial term which is responsible for generating a mass for the $U(1)$ field via the Stückelberg Mechanism, which is a standard mass generation mechanism in string theory according to which non-anomalous $U(1)$ vector fields may acquire masses. The details of how such a mass is generated are given in appendix A.

2.3 The total 4-dimensional action

We may now write down the complete expression for the three fields we are considering. This includes the total action for the gauge field A_μ living on the D3-brane (containing the DBI and WZ pieces), as well as the actions for the position coordinate r and the dilaton field ϕ . Using the results in appendix A and considering the brane to be moving in the radial direction only (generalisation to multi-field scenarios will be commented upon later), the final D3-brane action, in terms of the canonically normalised vector field $\mathcal{A}_\mu = A_\mu/\tilde{g}$ with $\tilde{g}^2 = T_3^{-1}(2\pi\alpha')^{-2}$ (see appendix A), is given by

$$S_{\text{D3}} = - \int d^4x \sqrt{-g} \left(h^{-1} \sqrt{\Lambda} - q h^{-1} + q \frac{m^2}{2} \mathcal{A}_\mu \mathcal{A}^\mu - q \frac{\mathcal{C}_0}{8} \epsilon^{\mu\nu\lambda\beta} \mathcal{F}_{\mu\nu} \mathcal{F}_{\lambda\beta} \right), \quad (2.8)$$

where

$$\begin{aligned} \Lambda = 1 + \frac{h e^{-\phi}}{2} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} + \frac{h^2 e^{-2\phi}}{8} (\mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} \mathcal{F}_{\gamma\delta} \mathcal{F}^{\gamma\delta} - 2 \mathcal{F}_{\alpha\beta} \mathcal{F}^{\beta\gamma} \mathcal{F}_{\gamma\delta} \mathcal{F}^{\delta\alpha}) + h \partial_\alpha \varphi \partial^\alpha \varphi \\ + 3 h l^2 (\partial_\alpha \varphi \partial^\alpha \varphi F_{\beta\gamma} F^{\beta\gamma} - 2 \partial_\alpha \varphi F^{\alpha\beta} \partial^\gamma \varphi F_{\gamma\beta}). \end{aligned}$$

In this expression we have introduced the canonically normalised (fixed) position field defined by $\varphi = \sqrt{T_3} r$ associated to the (radial) coordinate brane position r , and the corresponding warp factor is defined as $h(\varphi) = T_3^{-1} h(r)$. The dilaton dependent mass is given in string units by

$$m^2 = e^{-\phi} (2\pi)^4 \frac{M_s^2}{\mathcal{V}_6}, \quad (2.9)$$

where the dimensionless (warped) 6D volume is defined as $\mathcal{V}_6 = V_6 M_s^6$ (see appendix A). Furthermore $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita tensor, such that $\epsilon_{0123} = \sqrt{-g}$.

Coupling this action to four dimensional gravity, and including the necessary terms for an evolving dilaton as well as the potential for the brane's position, which will arise due to various effects such as fluxes and presence of other objects, we can write⁴

$$\begin{aligned} S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \\ - \int d^4x \sqrt{-g} \left[h^{-1} \sqrt{\Lambda} + V(\varphi) - q h^{-1} + q \frac{m^2}{2} \mathcal{A}_\mu \mathcal{A}^\mu \right], \quad (2.10) \end{aligned}$$

where $M_P^2 = 2 V_6 / [(2\pi)^7 \alpha'^4]$ is the Planck mass.

⁴We drop the topological term at this stage since C_0 is stabilised and therefore we may use the fact that $dA \wedge dA = d(A \wedge dA)$, hence this term constitutes a total derivative and thus will not give any effect in the cosmological evolution.

3. Stationary brane

We are now ready to study the cosmological implications of the $U(1)$ gauge field which lives on the D3-brane world volume. We start by considering the case in which the brane whose world volume hosts the vector field of interest is stationary in the internal space. Inflation is considered to be driven by a different D3-brane or any other working inflationary mechanism. Therefore our “curvaton brane” is just one of the D3-branes that may be present in the bulk at the time of inflation, for which $V(\varphi) = \text{constant}$, $\dot{\varphi} = 0$.

In general, non-trivial dilaton gradients may be sourced by D7-branes in the bulk. Such gradients are expected to be stabilised at the energies we are considering, however as mentioned in the Introduction, the dilaton may experience a displacement from its minimum without disturbing the geometry or the potential for the D3-brane that may be driving inflation. If such a displacement remains within the regime in which we can trust the weak coupling limit, we expect these approximations to hold. Therefore in what follows we will look at two possibilities for the dilaton: either it is fixed during inflation or it is able to evolve.

For a stationary D3-brane, the action in Eq. (2.10) then simplifies to

$$\begin{aligned}
S = & \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \\
& + \int d^4x \sqrt{-g} \left\{ h^{-1} \left[q - \sqrt{1 + \frac{h e^{-\phi}}{2} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} + \frac{h^2 e^{-2\phi}}{8} (\mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} \mathcal{F}_{\gamma\delta} \mathcal{F}^{\gamma\delta} - 2 \mathcal{F}_{\alpha\beta} \mathcal{F}^{\beta\gamma} \mathcal{F}_{\gamma\delta} \mathcal{F}^{\delta\alpha})} \right] \right. \\
& \left. - q \frac{m^2}{2} \mathcal{A}_\mu \mathcal{A}^\mu \right\}, \tag{3.1}
\end{aligned}$$

We now focus on the equations of motion for the vector field, derived from the action above. These are given by (from now on we take $q = 1$)

$$\begin{aligned}
G_{\mu\nu} = & \frac{\gamma_A}{2} \left(e^{-\phi} \mathcal{F}_{\mu\beta} \mathcal{F}_\nu{}^\beta + \frac{h e^{-2\phi}}{2} \mathcal{F}_{\mu\kappa} \mathcal{F}_\nu{}^\kappa \mathcal{F}^2 - h e^{-2\phi} \mathcal{F}_\nu{}^\sigma \mathcal{F}_{\sigma\delta} \mathcal{F}^{\delta\kappa} \mathcal{F}_{\kappa\mu} \right) \\
& + \frac{m^2 e^\phi}{2} \mathcal{A}_\mu \mathcal{A}_\nu + \frac{g_{\mu\nu} h^{-1}}{2} \left(1 - \gamma_A^{-1} - \frac{m^2}{2} \mathcal{A}_\mu \mathcal{A}^\mu \right), \tag{3.2}
\end{aligned}$$

$$\sqrt{-g} m^2 \mathcal{A}^\nu = \partial_\mu \left[\sqrt{-g} \gamma_A \left(e^{-\phi} \mathcal{F}^{\mu\nu} - h e^{-2\phi} \mathcal{M}^{\nu\mu} + \frac{h e^{-2\phi}}{2} \mathcal{N}^{\mu\nu} \right) \right], \tag{3.3}$$

where we have defined:

$$\gamma_A^{-1} = \sqrt{1 + \frac{he^{-\phi}}{2} \mathcal{F}^2 + \frac{h^2 e^{-2\phi}}{8} (\mathcal{F}^4 - 2\mathcal{F}_{\alpha\beta} \mathcal{F}^{\beta\gamma} \mathcal{F}_{\gamma\delta} \mathcal{F}^{\delta\alpha})}, \quad (3.4)$$

$$\mathcal{M}^{\nu\mu} = \mathcal{F}^{\nu\beta} \mathcal{F}_{\beta\gamma} \mathcal{F}^{\gamma\mu}, \quad (3.5)$$

$$\mathcal{N}^{\mu\nu} = \mathcal{F}^{\mu\nu} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} \quad (3.6)$$

and $G_{\mu\nu} = R_{\mu\nu} - \frac{g_{\mu\nu}}{2} R$ is the Einstein tensor. In a FRW universe, the four dimensional metric takes the usual form

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad (3.7)$$

where $a(t)$ is the scale factor. Moreover, we can expect inflation to homogenise the vector field, and therefore for the background solution

$$\partial_i \mathcal{A}_\mu = 0. \quad (3.8)$$

Using this condition, one can check that the factor γ_A associated with the vector field is given simply by

$$\gamma_A = \frac{1}{\sqrt{1 + \frac{he^{-\phi}}{2} \mathcal{F}^2}} = \frac{1}{\sqrt{1 - he^{-\phi} a^{-2} \dot{\mathcal{A}} \cdot \dot{\mathcal{A}}}}. \quad (3.9)$$

Moreover, the $\nu = t$ component of the vector field equation implies that

$$\mathcal{A}_t = 0, \quad (3.10)$$

and we thus have $\mathcal{A}_\mu = (0, \mathcal{A}(t))$. Using this, the $\nu = i$ component of the equation of motion becomes

$$\gamma_A \ddot{\mathcal{A}} + \gamma_A \dot{\mathcal{A}} \left(H + \frac{\dot{\gamma}_A}{\gamma_A} + \frac{\dot{f}}{f} \right) + \frac{m^2}{f} \mathcal{A} = 0, \quad (3.11)$$

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter, further we defined

$$f \equiv e^{-\phi}, \quad (3.12)$$

and we have used the fact that the \mathcal{M} and \mathcal{N} terms cancel each other in the background solution.

From the form of Eq. (3.11) we see that the effective mass $M \equiv \frac{m}{\sqrt{f}}$ is constant and given by

$$M = \sqrt{\pi} (2\pi)^5 \frac{M_P}{\mathcal{V}_6}. \quad (3.13)$$

As we demonstrate below, the desired behaviour of our system is attained when $f \propto a^2$. Solving Eqs. (3.11) and (3.9) in the case that $M \ll H$ and $e^{-\phi} \propto a^2(t)$, we

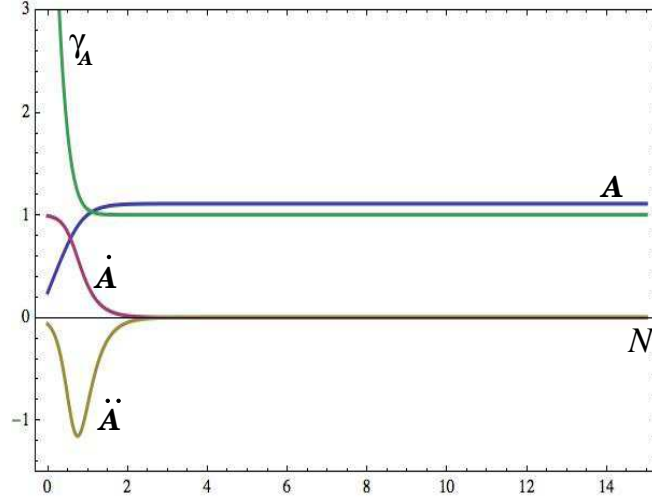


Figure 1: The qualitative behaviour for the vector background where blue = $\mathcal{A}(t)$, red = $\dot{\mathcal{A}}(t)$, yellow = $\ddot{\mathcal{A}}(t)$; green = $\gamma_A(t)$. In this figure, the horizontal axis measures the elapsing e-folds, while the vertical axis measures the value of $\gamma_A(t)$

obtain the behaviour of the background \mathcal{A}_μ and γ_A during the inflationary period. The results are plotted in Fig. 1, in which we simply indicate the qualitative behaviour of the vector background and its time derivatives. We see that the background soon freezes out at constant amplitude during inflation, while γ_A converges quickly to 1. This is in agreement with the findings of Ref. [9], when $f \propto a^2$.

3.1 Constant dilaton case

In this subsection we consider the case in which the dilaton field has already been stabilised once fluxes are introduced and remains so during the cosmological evolution. We explore the case of a rolling dilaton in the next section.

To focus purely on the non-standard vector terms for now, we simply take $e^\phi = e^{\phi_0} = g_s$. We then calculate the perturbations of the vector field during the cosmological evolution to see what sort of new terms arise. Therefore we perturb the vector field around the homogeneous value $\mathcal{A}_\mu(t)$ as follows:

$$\begin{aligned} \mathcal{A}_\mu(t, \mathbf{x}) &= \mathcal{A}_\mu(t) + \delta\mathcal{A}_\mu(t, \mathbf{x}) \quad \Rightarrow \\ \mathcal{A}_t(t, \mathbf{x}) &= \delta\mathcal{A}_t(t, \mathbf{x}) \quad \& \quad \mathcal{A}_\mu(t, \mathbf{x}) = \mathcal{A}_\mu(t) + \delta\mathcal{A}_\mu(t, \mathbf{x}). \end{aligned} \quad (3.14)$$

Then, the equation of motion for the temporal component $\nu = t$ becomes

$$a^2 m^2 \delta\mathcal{A}_t - \gamma_A \left[\nabla \cdot \delta\dot{\mathcal{A}} - \nabla^2 \delta\mathcal{A}_t \right] - h a^{-2} \gamma_A^3 \dot{\mathcal{A}} \cdot \nabla \left[\dot{\mathcal{A}} \cdot \delta\dot{\mathcal{A}} - \dot{\mathcal{A}} \cdot \nabla(\delta\mathcal{A}_t) \right] = 0. \quad (3.15)$$

The spatial component takes a more complicated form and is given by

$$\begin{aligned}
& -am^2\delta\mathcal{A} - a\gamma_A \left(\delta\ddot{\mathcal{A}} + H\delta\dot{\mathcal{A}} + \frac{\dot{\gamma}_A}{\gamma_A}\delta\dot{\mathcal{A}} \right) - ha^{-1}\gamma_A^3\dot{\mathcal{A}}\dot{\mathcal{A}} \cdot \left(\delta\ddot{\mathcal{A}} - H\delta\dot{\mathcal{A}} + 3\frac{\dot{\gamma}_A}{\gamma_A}\delta\dot{\mathcal{A}} \right) \\
& + a^{-1}\gamma_A\nabla^2\delta\mathcal{A} - ha^{-1}\gamma_A^3 \left\{ \dot{\mathcal{A}}(\ddot{\mathcal{A}} \cdot \delta\dot{\mathcal{A}}) + \ddot{\mathcal{A}}(\dot{\mathcal{A}} \cdot \delta\dot{\mathcal{A}}) - \dot{\mathcal{A}}[\ddot{\mathcal{A}} \cdot \nabla(\delta\mathcal{A}_t)] - \ddot{\mathcal{A}}[\dot{\mathcal{A}} \cdot \nabla(\delta\mathcal{A}_t)] \right\} \\
& + a\gamma_A\nabla \left(\frac{\dot{\gamma}_A}{\gamma_A}\delta\mathcal{A}_t - 2H\delta\mathcal{A}_t \right) + ha^{-1}\gamma_A^3\dot{\mathcal{A}}(\dot{\mathcal{A}} \cdot \nabla) \left(3\frac{\dot{\gamma}_A}{\gamma_A}\delta\mathcal{A}_t - 4H\delta\mathcal{A}_t + a^{-2}\nabla \cdot \delta\mathcal{A} \right) \\
& + h\gamma_A a^{-3} \left\{ -\dot{\mathcal{A}}(\dot{\mathcal{A}} \cdot \nabla)(\nabla \cdot \delta\mathcal{A}) + \dot{\mathcal{A}}\nabla^2(\dot{\mathcal{A}} \cdot \delta\mathcal{A}) - \dot{\mathcal{A}} \cdot \nabla[\nabla(\dot{\mathcal{A}} \cdot \delta\mathcal{A})] \right. \\
& \left. + (\dot{\mathcal{A}} \cdot \nabla)(\dot{\mathcal{A}} \cdot \nabla)\delta\mathcal{A} - (\dot{\mathcal{A}} \cdot \dot{\mathcal{A}})\nabla^2\delta\mathcal{A} + (\dot{\mathcal{A}} \cdot \dot{\mathcal{A}})\nabla(\nabla \cdot \delta\mathcal{A}) \right\} = 0. \tag{3.16}
\end{aligned}$$

We now pass to momentum space by performing a Fourier expansion of the perturbations as follows:

$$\delta\mathcal{A}_\mu(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \delta\mathcal{A}_\mu(t, \mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{x}). \tag{3.17}$$

Plugging this into (3.15) and (3.16), we can write the equations of motion for the transverse and longitudinal components as follows. Making the definitions:

$$\delta\mathcal{A}^\parallel \equiv \frac{\mathbf{k}(\mathbf{k} \cdot \delta\mathcal{A})}{k^2}, \quad \delta\mathcal{A}^\perp \equiv \delta\mathcal{A} - \delta\mathcal{A}^\parallel,$$

$$k^2 \equiv \mathbf{k} \cdot \mathbf{k}, \quad \gamma_A = \frac{1}{\sqrt{1 - h\left(\frac{\dot{\mathcal{A}}}{a}\right)^2}},$$

$$Q = \frac{h\gamma_A^3}{a^2} \left[\dot{\mathcal{A}}(\ddot{\mathcal{A}} \cdot \mathbf{k}) + \ddot{\mathcal{A}}(\dot{\mathcal{A}} \cdot \mathbf{k}) + \dot{\mathcal{A}}(\dot{\mathcal{A}} \cdot \mathbf{k}) \left(3\frac{\dot{\gamma}_A}{\gamma_A} - 4H \right) \right] + \gamma_A \mathbf{k} \left(\frac{\dot{\gamma}_A}{\gamma_A} - 2H \right),$$

$$R = (am)^2 - \gamma_A k^2 - \frac{h\gamma_A^3}{a^2} (\dot{\mathcal{A}} \cdot \mathbf{k})^2,$$

the transverse component becomes

$$\begin{aligned}
& \left\{ \partial_t^2 + \left(H + \frac{\dot{\gamma}_A}{\gamma_A} \right) \partial_t + \frac{m^2}{\gamma_A} + \left(\frac{k}{a} \right)^2 + \frac{h}{a^2} \left[\left(\frac{\dot{\mathcal{A}} \cdot \mathbf{k}}{a} \right)^2 - \dot{\mathcal{A}}^2 \left(\frac{k}{a} \right)^2 \right] \right\} \delta \mathcal{A}^\perp \\
& + h \left(\frac{\gamma_A}{a} \right)^2 \left\{ \dot{\mathcal{A}} (\dot{\mathcal{A}} \cdot \delta \dot{\mathcal{A}}^\perp) + \left[\dot{\mathcal{A}} \left(3 \frac{\dot{\gamma}_A}{\gamma_A} - H \right) + \ddot{\mathcal{A}} + \frac{Q}{R} (\dot{\mathcal{A}} \cdot \mathbf{k}) \right] \dot{\mathcal{A}} \cdot \delta \dot{\mathcal{A}}^\perp \right\} \\
& + \frac{h}{a^4} \left[\dot{\mathcal{A}} k^2 - (\dot{\mathcal{A}} \cdot \mathbf{k}) \mathbf{k} \right] \dot{\mathcal{A}} \cdot \delta \mathcal{A}^\perp + h \left(\frac{\gamma_A}{a} \right)^2 \dot{\mathcal{A}} (\ddot{\mathcal{A}} \cdot \delta \dot{\mathcal{A}}^\perp) = 0, \tag{3.18}
\end{aligned}$$

while the longitudinal component is:

$$\begin{aligned}
& \left\{ \partial_t^2 + \left[H + \frac{\dot{\gamma}_A}{\gamma_A} + \frac{\gamma_A}{R} \left(\frac{\dot{\gamma}_A}{\gamma_A} - 2H \right) \right] \partial_t + \frac{m^2}{\gamma_A} + \left(\frac{k}{a} \right)^2 + \frac{h}{a^2} \left(\frac{\dot{\mathcal{A}} \cdot \mathbf{k}}{a} \right)^2 \right\} \delta \mathcal{A}^\parallel \\
& + h \left(\frac{\gamma_A}{a} \right)^2 \dot{\mathcal{A}} (\dot{\mathcal{A}} \cdot \delta \dot{\mathcal{A}}^\parallel) + \left\{ h \left(\frac{\gamma_A}{a} \right)^2 \left[\dot{\mathcal{A}} \left(3 \frac{\dot{\gamma}_A}{\gamma_A} - H \right) + \ddot{\mathcal{A}} + \frac{Q}{R} (\dot{\mathcal{A}} \cdot \mathbf{k}) \right] \right. \\
& \left. + \frac{h \gamma_A^3}{R} \left[\ddot{\mathcal{A}} + \dot{\mathcal{A}} \left(3 \frac{\dot{\gamma}_A}{\gamma_A} - 4H \right) \right] \left(\frac{k}{a} \right)^2 \right\} \dot{\mathcal{A}} \cdot \delta \dot{\mathcal{A}}^\parallel \\
& + \frac{h}{a^2} \left[\dot{\mathcal{A}} \gamma_A^2 \left(\frac{k}{a} \right)^2 - \frac{(\dot{\mathcal{A}} \cdot \mathbf{k}) \mathbf{k}}{a^2} \right] \dot{\mathcal{A}} \cdot \delta \mathcal{A}^\parallel + h \gamma_A^2 \dot{\mathcal{A}} \left[\frac{1}{a^2} + \frac{\gamma_A}{R} \left(\frac{k}{a} \right)^2 \right] (\ddot{\mathcal{A}} \cdot \delta \dot{\mathcal{A}}^\parallel) = 0. \tag{3.19}
\end{aligned}$$

At this point we can compare these equations to the standard kinetic term case [8, 9]. In fact, the first lines in equations (3.18) and (3.19) have the same form as the standard case in which the kinetic term for the vector field has the form $f \mathcal{F}^2$. In our present case, $f \rightarrow \gamma_A$ and we get an extra term proportional to $\dot{\mathcal{A}}^2 k^2$. The other lines in these equations are new and appear due to the third order terms \mathcal{M}, \mathcal{N} in the equations of motion [cf. Eqs. (3.5) and (3.6)]. In principle, these terms cannot be neglected once we consider $\gamma_A > 1$.

We can get some insight into these equations by solving (3.9) and (3.11) for a constant dilaton during the inflationary period. We then see that once again, the background \mathcal{A}_μ freezes out at constant amplitude, and the behaviour of the system quickly converges to a situation in which $\gamma_A \approx 1$ (see Fig. 2). Therefore, the new

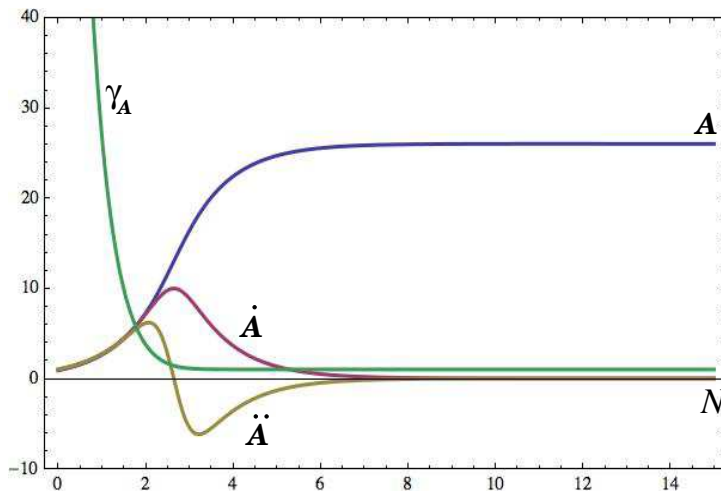


Figure 2: The qualitative behaviour for the vector background where blue = $\mathcal{A}(t)$, red = $\dot{\mathcal{A}}(t)$, yellow = $\ddot{\mathcal{A}}(t)$; green = $\gamma_A(t)$. In this figure, the horizontal axis measures the elapsing e-folds, while the vertical axis measures the value of $\gamma_A(t)$

terms in (3.18) and (3.19) are suppressed at late times when \mathcal{A}_μ is approximately constant, however the effects of these terms may be important for the scales that leave the horizon at early times. For the time being, we concentrate on the case in which $\gamma_A \sim 1$ and therefore we recover the standard form for the kinetic term of vector field.

3.2 Varying dilaton case

In this section we consider the case in which $\gamma_A \sim 1$ but allow the dilaton to evolve due to some effect which can move it away from its minimum. For the moment, we do not worry about a concrete realisation of such a scenario, but consider the possible implications of this case.

We begin by outlining the conditions which should be placed on the evolution of the dilaton in order to achieve a scale-invariant spectrum of vector perturbations. It was clearly demonstrated in Ref. [9] that, for a light vector field evolving during quasi-de Sitter expansion with a gauge kinetic function $f \propto a^\alpha$ and mass $m \propto a^\beta$, the power spectrum of vector perturbations is exactly scale invariant when $\alpha = -1 \pm 3$ and $\beta = 1$, as long as the vector field remains light when the cosmological scales exit the horizon. In the case that $f \propto a^{-4}$ (and $m \propto a$), the power spectra for the transverse and longitudinal components of the vector perturbations can become roughly equal if the vector field becomes heavy by the end of inflation. This allows for (approximately) isotropic particle production and entails that the vector field can provide the dominant contribution to the curvature perturbation, which is known to be predominantly isotropic. In the other case, *i.e.* $f \propto a^2$ (and $m \propto a$), the power spectrum for the transverse component of the vector perturbations is much larger

than that of the longitudinal component, therefore particle production is strongly anisotropic and the dominant contribution to the curvature perturbation must come from some other, isotropic source, *e.g.* a scalar field.⁵ However, the important point is that, in this case, the vector field can contribute measureable statistical anisotropy.

In our model, $f \propto e^{-\phi}$ and $m \propto e^{-\phi/2} = \sqrt{f}$ [cf. Eqs. (2.9) and (3.12)], such that if we demand $m \propto a$ we must have $f \propto a^2$, *i.e.* we arrive at an explicit realisation of the latter of the above two possibilities which leads to scale invariance of the vector perturbations as shown in Ref. [9], as long the D-brane vector field is light and evolving during quasi-de Sitter expansion, and, crucially, as long as $e^{-\phi} \propto a^2$.

This dependence of the dilaton on the scale factor implies that $\dot{\phi} = -2HM_P$, which suggests that the variation of the dilaton over ΔN e-foldings is $\Delta\phi = 2\Delta NM_P$. It also suggests that the appropriate variation can be achieved by a linear potential, since the Klein-Gordon equation gives $V'(\phi) \simeq 6H^2M_P$, where the prime denotes derivative with respect to the dilaton.⁶ Any form of scalar potential away from extrema and discontinuities can be Taylor expanded to an approximately linear potential, as long as it is flat enough. However, the approximation has to remain valid during the period when the cosmological scales exit the horizon $\Delta N \simeq 10$. Also, the dilaton scalar potential has to have a subdominant contribution to the total energy budget, if it is not to drive inflation itself.

The potential for the dilaton is given by the sum of exponential terms plus a possible constant contribution, and in a suitable regime where the exponentials can be approximated by a linear term, the required dependence on the scale factor can be attained, as explained above. Since the dilaton needs to roll for a prolonged period of time, its effective potential during this time has to be sufficiently flat. One possibility to arrange for such a behaviour is by identifying an almost flat direction in the potential for the moduli, which is mostly dominated by the dilaton. Such a direction would need to remain adequately flat for the required duration of the dilaton's modulation of the vector curvaton, which, as shown below, is less than the total e-foldings of inflation. In what follows we assume that these properties for the dilaton potential can be achieved and leave for future work a study of a more concrete realisation of them.

We now calculate the equations of motion for the vector perturbations in this case. In view of Eqs. (2.9) and (3.12), the equations for the perturbations simplify to

⁵This is so even for the case in which $f \propto a^{-4}$ (and $m \propto a$) if the vector field remains light until the end of inflation [9].

⁶Here we ignored the vector field backreaction because \mathcal{A} is frozen.

$$\frac{(am)^2}{f}\delta\mathcal{A}_0 - \nabla^2\delta\mathcal{A}_0 + \nabla\cdot\delta\dot{\mathcal{A}} = 0, \quad (3.20)$$

$$\frac{m^2}{f}\delta\mathcal{A} + H\delta\dot{\mathcal{A}} + \delta\ddot{\mathcal{A}} + \left(\delta\dot{\mathcal{A}} - \nabla\delta\mathcal{A}_0\right)\frac{\dot{f}}{f} - a^{-2}\nabla^2\delta\mathcal{A} + 2\left(H + \frac{\dot{m}}{m}\right)\nabla\delta\mathcal{A}_0 = 0. \quad (3.21)$$

Moving to Fourier space using Eq. (3.17) as before, the equations for the perturbations (3.20) become

$$\delta\mathcal{A}_0 + \frac{i\partial_t(\mathbf{k}\cdot\delta\mathcal{A})}{\left[k^2 + \frac{(am)^2}{f}\right]} = 0, \quad (3.22)$$

$$\frac{m^2}{f}\delta\mathcal{A} + \left(H + \frac{\dot{f}}{f}\right)\delta\dot{\mathcal{A}} + \delta\ddot{\mathcal{A}} + \left(\frac{k}{a}\right)^2\delta\mathcal{A} + \left(2H + 2\frac{\dot{m}}{m} - \frac{\dot{f}}{f}\right)\frac{\partial_t(\mathbf{k}\cdot\delta\mathcal{A})}{\left[k^2 + \frac{(am)^2}{f}\right]} = 0. \quad (3.23)$$

We can now compute the equations for the longitudinal and transverse components as before. Using the relations:

$$\delta\mathcal{A}^{\parallel} \equiv \frac{\mathbf{k}(\mathbf{k}\cdot\delta\mathcal{A})}{k^2}, \quad \delta\mathcal{A}^{\perp} \equiv \delta\mathcal{A} - \delta\mathcal{A}^{\parallel} \quad (3.24)$$

the equations become:

$$\left[\frac{m^2}{f} + \left(H + \frac{\dot{f}}{f}\right)\partial_t + \partial_t^2 + \left(\frac{k}{a}\right)^2\right]\delta\mathcal{A}^{\perp} = 0 \quad (3.25)$$

$$\left[\frac{m^2}{f} + \left(H + \frac{\dot{f}}{f}\right)\partial_t + \left(2H + 2\frac{\dot{m}}{m} - \frac{\dot{f}}{f}\right)\frac{\left(\frac{k}{a}\right)^2\partial_t}{\left(\frac{k}{a}\right)^2 + \frac{m^2}{f}} + \partial_t^2 + \left(\frac{k}{a}\right)^2\right]\delta\mathcal{A}^{\parallel} = 0, \quad (3.26)$$

which are identical to those obtained in Ref. [8].

At this point we define the spatial components of the canonically normalised, physical (as opposed to comoving) vector field as follows:

$$\mathbf{W} \equiv \sqrt{f}\mathcal{A}/a. \quad (3.27)$$

Moving to Fourier space, we expand the perturbations of the physical vector field, $\delta\mathbf{W}$, as

$$\delta\mathbf{W}(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \delta\mathcal{W}(t, \mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (3.28)$$

The field equations for the spatial vector perturbations (3.25) then become

$$\left[\partial_t^2 + 3H\partial_t + M^2 + \left(\frac{k}{a} \right)^2 \right] \delta \mathbf{w}^\perp = 0, \quad (3.29)$$

and

$$\left\{ \partial_t^2 + \left[3H + 2H \frac{\left(\frac{k}{a} \right)^2}{\left(\frac{k}{a} \right)^2 + M^2} \right] \partial_t + M^2 + \left(\frac{k}{a} \right)^2 \right\} \delta \mathbf{w}^\parallel = 0, \quad (3.30)$$

which are the same as those which were found in Ref. [9].

4. Particle production

Gravitational particle production of a vector field proceeds analogously to that of a scalar field: vacuum fluctuations are exponentially stretched by the expansion and become frozen on superhorizon scales, imprinting their spectrum on the homogeneous background. As a realisation of Hawking radiation in a de Sitter background, it can be shown that this process gives rise to the appearance of real particles, which are then interpreted as having been created by the changing gravitational field [5].

From now on we regard the perturbations of the vector field as quantum fields $\delta \hat{\mathbf{W}}$, whose time-dependent mode functions $w(t, \mathbf{k})$ solve the linear equations of motion (3.29) and (3.30) given above. We seek late time solutions to these equations, where the integration constants will be determined by matching the solution to the Bunch-Davis vacuum at early times. We now solve the equations of motion and determine the resulting statistical anisotropy in the curvature perturbation, closely following the analysis presented in Ref. [9].

4.1 The transverse component

Starting with the equation of motion for the transverse mode functions which, due to the invariance of the theory under parity, may be written as $w_L = w_R \equiv w_{L,R}$, we obtain

$$\ddot{w}_{L,R} + 3H\dot{w}_{L,R} + \left[M^2 + \left(\frac{k}{a} \right)^2 \right] w_{L,R} = 0. \quad (4.1)$$

This equation matches the equation of motion solved by the mode function of a scalar field evolving during quasi-de Sitter expansion, which is well-known to produce a scale invariant spectrum of super-horizon perturbations as long as $M \ll H$ while the cosmological scales exit the horizon. In order for the vector field perturbations to obtain a scale invariant superhorizon spectrum, it is therefore a necessary condition that the physical mass of the vector field is light compared to the Hubble scale at horizon exit.

To ensure that the condition $M \ll H$ is met when the cosmological scales exit the horizon, we assume that the field remains effectively massless for the entire duration of the inflationary period. We then compute the power spectrum for the superhorizon vector perturbations. In Ref. [9] it is shown that even if this condition is violated before inflation ends, but after the cosmological scales have left the horizon, the dependence of the power spectrum on scale is unaffected for the cosmological scales. In what follows we will show that meeting the requirement $M \ll H$ at horizon exit places a bound on the size of the compact volume.

Taking the field to be effectively massless during inflation, $M \ll k/a$, the behaviour of Eq. (4.1) for sub-Hubble and super-Hubble modes, $k \gg aH$ and $k \ll aH$ respectively, can be read off immediately. In the case of sub-Hubble modes, the friction term is subdominant and the equation reduces to that of a harmonic oscillator with a wave number that is diluted by the scale factor. In the case of super-Hubble modes, the gradient term is subdominant and the resulting equation is solved by a constant. This captures the fact that the vacuum fluctuations will oscillate until horizon exit, at which point they become effectively frozen at constant amplitude.

By defining $r \equiv aM/k$ (where $r \ll 1$ during inflation for an effectively massless field $M \ll k/a$), the equation of motion for the transverse mode functions may be written as

$$\ddot{w}_{L,R} + 3H\dot{w}_{L,R} + \left(\frac{k}{a}\right)^2 (1 + r^2)w_{L,R} = 0. \quad (4.2)$$

This equation may be solved exactly. To obtain the integration constants, one applies the Bunch-Davis vacuum boundary condition, which holds for very early times at which the scales are well within the horizon. At such times, the effects of the expanding space are not yet apparent and one may consider the theory in Minkowski spacetime. The vacuum boundary condition takes the form

$$\lim_{(k/aH) \rightarrow +\infty} w_{L,R} = \frac{a^{-1}}{\sqrt{2k}} e^{ik/aH} \quad \Rightarrow \quad (4.3)$$

$$w_{L,R} = \frac{a^{-3/2}}{2} \sqrt{\frac{\pi}{H}} e^{i(\pi/2)[\nu+(1/2)]} H_\nu^{(1)}(k/aH), \quad (4.4)$$

where $H_\nu^{(1)}$ is the Hankel function of the first kind and $\nu = 3/2$.

To obtain the power spectrum of superhorizon vector perturbations, we must evaluate this solution at late times, $(k/aH) \rightarrow 0^+$. The dominant term in the solution for late times is given by

$$\lim_{(k/aH) \rightarrow 0^+} w_{L,R} = -\frac{ia^{-3/2}}{2\Gamma(1-\nu)} \sqrt{\frac{\pi}{H}} e^{i(\pi/2)[\nu+(1/2)]} \left(\frac{k}{aH}\right)^{-\nu}. \quad (4.5)$$

The power spectrum is then given by

$$\mathcal{P}_{L,R} \equiv \frac{k^3}{2\pi^2} \left| \lim_{(k/aH) \rightarrow 0^+} w_{L,R} \right|^2 = \frac{4\pi}{[\Gamma(1-\nu)]^2} \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{2aH}\right)^{3-2\nu}. \quad (4.6)$$

For our case in which $\nu = 3/2$, this simplifies to

$$\mathcal{P}_{L,R} = \left(\frac{H}{2\pi} \right)^2. \quad (4.7)$$

which is clearly scale invariant.

4.2 The longitudinal component

We now compute the power spectrum for the longitudinal mode function $w_{||}$, for which the equation of motion takes the form

$$\ddot{w}_{||} + \left[3H + 2H \frac{\left(\frac{k}{a}\right)^2}{\left(\frac{k}{a}\right)^2 + M^2} \right] \dot{w}_{||} + \left[M^2 + \left(\frac{k}{a}\right)^2 \right] w_{||} = 0. \quad (4.8)$$

In terms of r defined above, this is recast as

$$\ddot{w}_{||} + \left(3 + \frac{2}{1+r^2} \right) H \dot{w}_{||} + \left(\frac{k}{a} \right)^2 (1+r^2) w_{||} = 0. \quad (4.9)$$

As before, this equation may be solved exactly, where the integration constants are found by matching the solution to the vacuum at early times. However, the vacuum condition applied in the case of the transverse components must now be multiplied by a Lorentz boost factor γ , in order to transform from the frame in which the momentum $\mathbf{k} = 0$ to a frame in which $\mathbf{k} \neq 0$, such that one might obtain a distinction between longitudinal and transverse components. The vacuum boundary condition therefore becomes,

$$\lim_{(k/aH) \rightarrow +\infty} w_{||} = \gamma \frac{a^{-1}}{\sqrt{2k}} e^{ik/aH}, \quad (4.10)$$

where $\gamma = E/M = \sqrt{1+1/r^2}$. The solution is then given by

$$w_{||} = \frac{a^{-3/2}}{r} \sqrt{\frac{\pi}{4H}} \frac{e^{-i(\pi/2)[\mu-(3/2)]}}{\sin(\pi\mu)} \left[J_{\mu}(k/aH) - e^{i\pi\mu} J_{-\mu}(k/aH) \right], \quad (4.11)$$

where $\mu = 5/2$ and J_{μ} is a Bessel function of the first kind.

The late time behaviour of this solution is given by

$$\lim_{(k/aH) \rightarrow 0^+} w_{||} = -\frac{a^{-3/2}}{\Gamma(1-\mu)} \sqrt{\frac{\pi}{H}} \frac{e^{-i(\pi/2)[\mu-(3/2)]}}{\sin(\pi\mu)} \left(\frac{H}{M} \right) \left(\frac{k}{2aH} \right)^{1-\mu}, \quad (4.12)$$

from which we obtain the superhorizon power spectrum,

$$\mathcal{P}_{||} \equiv \frac{k^3}{2\pi^2} \left| \lim_{(k/aH) \rightarrow 0^+} w_{||} \right|^2 = \frac{16\pi}{\sin^2(\pi\mu)[\Gamma(1-\mu)]^2} \left(\frac{H}{M} \right)^2 \left(\frac{H}{2\pi} \right)^2 \left(\frac{k}{2aH} \right)^{5-2\mu}. \quad (4.13)$$

Setting $\mu = 5/2$, we see that once again we arrive at a scale invariant spectrum,

$$\mathcal{P}_{\parallel} = 9 \left(\frac{H}{M} \right)^2 \left(\frac{H}{2\pi} \right)^2. \quad (4.14)$$

Comparing the results for the power spectra of the transverse and longitudinal mode functions, we see that, for $M \ll H$,

$$\mathcal{P}_{\parallel} \gg \mathcal{P}_{L,R}. \quad (4.15)$$

The above results are in agreement with Ref. [9] and they are central to the claim that a vector field modulated by an evolving dilaton field can contribute measurable statistical anisotropy to the curvature perturbation, as we will now demonstrate.

5. Statistical Anisotropy

Historically, statistical anisotropy due to the contribution of vector field perturbations to the curvature perturbation was first studied in the context of the inhomogeneous end of inflation mechanism [19] (see also Ref. [20]). However, the first comprehensive, mechanism independent, study of the effect of vector field perturbations onto ζ and the resulting statistical anisotropy in the spectrum and bispectrum is presented in Ref. [7]. In the present work, we focus on the vector curvaton mechanism, which has the advantage of not being restrictive on the inflation model as well as keeping the inflaton and vector curvaton sectors independent (they can correspond to two different branes for example).⁷

5.1 The power spectrum

We assume that the curvature perturbation ζ receives contributions from both the scalar inflaton field as well as the vector field. In terms of its isotropic and anisotropic contributions, the power spectrum of ζ may be written as [7]

$$\mathcal{P}_{\zeta}(\mathbf{k}) = \mathcal{P}_{\zeta}^{\text{iso}}(k)[1 + g(\hat{\mathbf{N}}_A \cdot \hat{\mathbf{k}})^2], \quad (5.1)$$

where $\mathcal{P}_{\zeta}^{\text{iso}}(k)$ is the dominant isotropic contribution, $\hat{\mathbf{k}} \equiv \mathbf{k}/k$ and $\hat{\mathbf{N}}_A \equiv \mathbf{N}_A/N_A$ are unit vectors in the directions \mathbf{k} and \mathbf{N}_A respectively, and g quantifies the degree of statistical anisotropy in the spectrum. The components of \mathbf{N}_A are given by $N_A^i \equiv \partial N / \partial W_i$ where $N_A \equiv |\mathbf{N}_A|$ and N gives the number of the remaining e-foldings of inflation. The degree of statistical anisotropy in the spectrum may be defined in terms of the various power spectra that contribute to \mathcal{P}_{ζ} as follows [7]

$$g \equiv N_A^2 \frac{\mathcal{P}_{\parallel} - \mathcal{P}_{L,R}}{\mathcal{P}_{\zeta}^{\text{iso}}}. \quad (5.2)$$

⁷Indirectly, statistical anisotropy in ζ can also be generated by considering a mild anisotropisation of the inflationary expansion, due to the presence of a vector boson field condensate. In this case, it is the perturbations of the inflaton scalar field which are rendered statistically anisotropic [21].

It is clear that, for the case at hand in which $\mathcal{P}_{\parallel} \gg \mathcal{P}_{L,R}$, the degree of statistical anisotropy can be non-negligible. The CMB data provide no more than a weak upper bound on g , which allows statistical anisotropy as much as 30% [22]. The forthcoming observations of the Planck satellite will reduce this bound down to 2% if statistical anisotropy is not observed [23]. This means that, for statistical anisotropy to be observable in the near future, g must lie in the range

$$0.02 \leq g \leq 0.3. \quad (5.3)$$

Thus, the spectrum is predominantly isotropic but not to a large degree. To avoid generating an amount of statistical anisotropy that is inconsistent with the observational bounds, we then require that the dominant contribution to the curvature perturbation comes from the scalar inflaton. Thus, the density parameter of the vector field is $\Omega_A \equiv \rho_A/\rho \ll 1$, where ρ_A and ρ are the densities of the vector field and of the Universe respectively. Therefore, in this case, the contribution of the vector field to ζ serves to imprint statistical anisotropy at a measurable level.

Let us attempt to quantify this contribution in the case of our D-brane vector curvaton model. In Ref. [7] (see also Ref. [10]) it was shown that, for the vector curvaton, when $\Omega_A \ll 1$ we have

$$N_A \simeq \frac{1}{2} \frac{\Omega_A}{W}, \quad (5.4)$$

where $W = |\mathbf{W}|$ is the modulus of the physical vector field. In view of the above, Eq. (5.2) can be recast as

$$\sqrt{g} \sim \frac{\Omega_A}{\zeta} \frac{\delta W}{W}, \quad (5.5)$$

where we have used that $\mathcal{P}_{\zeta}^{\text{iso}} \approx \zeta^2$, since the curvature perturbation is predominantly isotropic, and also that

$$\delta W \sim \sqrt{\mathcal{P}_{\parallel}} \sim H_*^2/M, \quad (5.6)$$

since $\mathcal{P}_{\parallel} \gg \mathcal{P}_{L,R}$ and we have employed Eq. (4.14), with H_* denoting the Hubble scale during inflation.

The contribution of the vector field to ζ is finalised at the time of decay of the vector curvaton, since until then it is evolving with time. Thus, we need to evaluate the above at the time of the vector curvaton decay, which we denote by ‘dec’. In Ref. [9] it was shown that

$$\Omega_A^{\text{dec}} \sim \Omega_A^{\text{end}} \left(\frac{\Gamma}{\Gamma_A} \right)^{1/2} \min \left\{ 1, \frac{M}{H_*} \right\}^{2/3} \min \left\{ 1, \frac{M}{\Gamma} \right\}^{-1/6}, \quad (5.7)$$

where Γ and Γ_A denote the decay rates of the inflaton and the vector curvaton fields respectively, and ‘end’ denotes the end of inflation.

During inflation, as long as the dilaton is varying and $f \propto a^2$ and $m \propto a$, the vector curvaton remains frozen with

$$W = W_* = \sqrt{f}\mathcal{A}/a \propto \mathcal{A} \simeq \text{constant} , \quad (5.8)$$

where we have considered Eq. (3.27). However, the dilaton is not expected to roll throughout the remaining 50-60 e-foldings of inflation, after the cosmological scales exit the horizon. Instead, being a spectator field, it will, most probably, stop rolling N_x e-foldings before the end of inflation. After this the modulation of f and m ceases and we have $f \rightarrow 1$. While the mass of the physical vector curvaton $M = m/\sqrt{f}$ remains constant, this is not so for W . Indeed, taking $f = 1$, it is easy to see that \mathcal{A} remains frozen. Thus, in view of Eq. (3.27), we find

$$W \propto 1/a . \quad (5.9)$$

It is clear that the same is also true of the vector field perturbation, *i.e.* $\delta W \propto 1/a$. Putting the above together we can estimate the value of Ω_A^{end} as follows:

$$\Omega_A^{\text{end}} \sim e^{-2N_x} \left(\frac{M}{H_*} \right)^2 \left(\frac{W_*}{M_P} \right)^2 , \quad (5.10)$$

where we used the Friedman equation $\rho = 3(H_*M_P)^2$ and also that, while the dilaton is varying, the density of the frozen vector curvaton remains constant with $\rho_A \sim M^2 W_*^2$ [9].

Combining the above with Eqs. (5.5), (5.6) and (5.7) we obtain

$$\sqrt{g} \sim \zeta^{-1} e^{-2N_x} \frac{H_* W_*}{M_P^2} \left(\frac{M}{H_*} \right)^{5/3} \min \left\{ 1, \frac{M}{\Gamma} \right\}^{-1/6} \left(\frac{\Gamma}{\Gamma_A} \right)^{1/2} , \quad (5.11)$$

where ‘*’ denotes the epoch when the cosmological scales leave the horizon and we considered that $M \ll H_*$ for particle production of the vector curvaton to take place.

5.2 The bispectrum

Now, let us consider the bispectrum. As is well known, the bispectrum of the curvature perturbation is a measure of the non-Gaussianity in ζ since it is exactly zero for Gaussian curvature perturbation. This non-Gaussianity is quantified by the so-called non-linearity parameter f_{NL} , which connects the bispectrum with the power spectra. When we have a contribution of a vector field to ζ , f_{NL} can be statistically anisotropic [7, 24]. In the case of the vector curvaton with a varying kinetic function and mass, as is the case in our model, it was shown in Ref. [9] that non-Gaussianity is predominantly anisotropic. This means that, if non-Gaussianity is indeed observed without a strong angular modulation on the microwave sky, our model will be excluded from explaining the dominant contribution to the non-Gaussian signal.

The value of f_{NL} depends on the configuration of the three momentum vectors which are used to define the bispectrum. In Ref. [9] it was demonstrated that statistical anisotropy is strongest in the so-called equilateral configuration, where the three momentum vectors are of equal magnitude. In this case,

$$\frac{6}{5}f_{\text{NL}}^{\text{eq}} = \frac{2g^2}{\Omega_A^{\text{dec}}} \left(\frac{M}{3H_*} \right)^4 \left[1 + \frac{1}{8} \left(\frac{3H_*}{M} \right)^4 \hat{W}_\perp^2 \right], \quad (5.12)$$

where \hat{W}_\perp is the modulus of the projection of the unit vector $\hat{\mathbf{W}} \equiv \mathbf{W}/W$ onto the plane determined by the three momentum vectors which define the bispectrum. From the above, we see that the amplitude of the modulated $f_{\text{NL}}^{\text{eq}}$ is [9]

$$\|f_{\text{NL}}^{\text{eq}}\| = \frac{5}{24} \frac{g^2}{\Omega_A^{\text{dec}}}, \quad (5.13)$$

while the degree of statistical anisotropy in non-Gaussianity is

$$\mathcal{G} = \frac{1}{8} \left(\frac{3H_*}{M} \right)^4 \gg 1, \quad (5.14)$$

which demonstrates that non-Gaussianity is predominantly anisotropic.

Combining Eqs. (5.11) and (5.13) we can eliminate the dependence on W_* and obtain

$$\|f_{\text{NL}}^{\text{eq}}\| \sim \frac{5}{24} g\zeta^{-2} e^{-2N_*} \left(\frac{H_*}{M_P} \right)^2 \left(\frac{M}{H_*} \right)^{2/3} \min \left\{ 1, \frac{M}{\Gamma} \right\}^{-1/6} \left(\frac{\Gamma}{\Gamma_A} \right)^{1/2}. \quad (5.15)$$

To proceed we note that

$$\frac{H_*}{M_P} < \frac{\delta W}{W} < 1, \quad (5.16)$$

where the upper bound is to ensure that our perturbative approach remains valid and the lower bound is due to the requirement that $\Omega_A < 1$, *i.e.* the vector field does not dominate the Universe at any stage. Indeed, since the ratio $\delta W/W$ remains constant throughout the evolution of the vector field, we find

$$\frac{\delta W}{W} \approx \left. \frac{\delta W}{W} \right|_* \sim \frac{H_*^2}{MW_*} \sim \frac{H_*}{M_P} \frac{1}{\sqrt{\Omega_{A*}}}, \quad (5.17)$$

where $\Omega_{A*} = (\rho_A/\rho)_*$. Employing Eq. (5.16), Eq. (5.11) gives

$$g\zeta^2 e^{4N_*} \left(\frac{M_P}{H_*} \right)^2 \frac{\Gamma_A}{\Gamma} < \left(\frac{M}{H_*} \right)^{4/3} \min \left\{ 1, \frac{M}{\Gamma} \right\}^{-1/3} < g\zeta^2 e^{4N_*} \left(\frac{M_P}{H_*} \right)^4 \frac{\Gamma_A}{\Gamma}. \quad (5.18)$$

Using this, Eq. (5.15) gives

$$\frac{5}{24} \frac{g^{3/2}}{\zeta} \frac{H_*}{M_P} < \|f_{\text{NL}}^{\text{eq}}\| < \frac{5}{24} \frac{g^{3/2}}{\zeta}. \quad (5.19)$$

From Eq. (5.3) the above suggests that

$$12 \leq \|f_{\text{NL}}^{\text{eq}}\|_{\text{max}} \leq 713, \quad (5.20)$$

where $\zeta = 4.8 \times 10^{-5}$ is the observed curvature perturbation. Thus, we see that, if the generated statistical anisotropy in the spectrum is observable then non-Gaussianity has also a good chance of being observable, especially since the upper bound in the above is already excessive and violates the observational constraints, $-214 < f_{\text{NL}}^{\text{eq}} < 266$ [1].

5.3 After inflation and reheating

To explore the observational consequences of our scenario we need to consider the evolution of the Universe after the end of inflation. Once the expansion rate has dropped sufficiently such that $H(t)$ becomes of order M , the vector field condensate begins quasi-harmonic coherent oscillations and produces curvaton quanta. It has been shown in Refs. [4, 9] that the energy density and average pressure of the field during the oscillations scale as $\rho_A \propto a^{-3}$ and $\bar{p} \approx 0$ respectively, *i.e.* the field behaves as pressureless isotropic matter. After inflation, we assume that $f = 1$ and $m = M$. Therefore the action for the vector field which is minimally coupled to gravity becomes

$$S = M_P^2 \int \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{4M_P^2} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - \frac{M^2}{2M_P^2} \mathcal{A}_\mu \mathcal{A}^\mu \right) d^4x. \quad (5.21)$$

We can make a lower bound estimate on the decay rate of the curvaton field quanta based on gravitational decay, for which the decay rate is given by

$$\Gamma_A \sim \frac{M^3}{M_P^2}. \quad (5.22)$$

The decay products of the vector curvaton are much lighter degrees of freedom which are, therefore, relativistic.

The physical mass of the vector field in terms of the Planck mass is given in Eq. (3.13), which we quote here:

$$M = \frac{(2\pi)^5 \sqrt{\pi}}{\mathcal{V}_6} M_P. \quad (5.23)$$

This should be compared to the inflationary Hubble scale

$$H = \frac{1}{M_P} \sqrt{\frac{V_0}{3}}, \quad (5.24)$$

where $V_0^{1/4}$ is the energy scale of inflation. Therefore, in order to obtain $M \ll H$ we require

$$V_0^{1/2} \gg \frac{(2\pi)^5 \sqrt{3\pi}}{\mathcal{V}_6} M_P^2. \quad (5.25)$$

We can estimate the scale of inflation for slow roll and DBI scalar inflation in order to obtain a bound on the compact volume.

- In a slow roll inflationary scenario, the CMB observations suggest [25]

$$V_0^{1/4} = 0.027\epsilon^{1/4}M_P, \quad (5.26)$$

where ϵ is the slow-roll parameter. Assuming that ϵ is not tiny (*e.g.* taking $\epsilon > 10^{-5}$) we obtain $V_0^{1/4} \sim 10^{16}$ GeV. Thus, for $M \ll H$ to be fulfilled, we require that the size of the dimensionless volume \mathcal{V}_6 is around $10^8 - 10^9$ or larger, in which case the physical mass of the vector field is $M < 10^{14}$ GeV. For such a physical mass, Eq. (5.22) suggests $\Gamma_A < 10^6$ GeV. The temperature at the time of the vector curvaton decay is $T_{\text{dec}} \sim \sqrt{M_P \Gamma_A}$. It is easy to see that the decay occurs before Big Bang Nucleosynthesis if $M > 10$ TeV.

- If inflation is instead driven by a DBI scenario, an equivalent expression to (5.26) can be found (see for example [26]) to be

$$V_0^{1/2} = 0.03 \tilde{h}^{-1/4} M_P^2, \quad (5.27)$$

where $\tilde{h} = h M_P^4$ is a dimensionless warp factor. For a GUT inflationary scale, which is consistent with DBI inflation, we require $\tilde{h}^{1/4} \sim 10^3$, which can be achieved near the tip of the throat, where the warp factor is larger. Using this, we again obtain a limit on the 6D compact volume of the same order as above, and thus the same results follow. Moreover, if inflation happens in a DBI fashion, a second source for large non-Gaussianities of the equilateral shape is generated [15].

What about the inflaton decay, which reheats the Universe (since the vector curvaton is always subdominant)? In the case where the vector field brane is static, inflation is driven by some other sector. Possibly, this is another brane undergoing motion along the warped throat that may be either of the slow roll or DBI variety. The end of inflation takes place when the inflaton brane approaches the tip of the throat, where there is an IR cutoff which allows the brane to reach the origin at finite time and oscillate around the minimum of the potential [15, 27, 28]. In this case, the inflaton decay rate Γ depends on the couplings of the inflaton to standard model particles, to which it decays. A more dramatic end of inflation can take place if the inflaton brane meets and annihilates with the antibrane located at the end of the tip. This is a complex process which occurs via a cascade that begins with a gas of closed strings, followed by Kaluza-Klein modes and eventually standard model particles [29]. In order to illustrate this scenario, we simply assume that all of the initial vacuum energy of the branes is converted into radiation [30]. This would lead to almost instantaneous reheating (prompt reheating) with a reheating temperature $T_{\text{reh}} \sim V_0^{1/4}$, which implies $\Gamma \sim H_*$.

The above possibilities may also arise in the case that our vector curvaton field is living on a moving brane, which can indeed be the inflaton brane, as will be

subsequently discussed. However, in this case, prompt reheating by annihilation is not possible because we need the vector field to survive after the end of inflation in order to play the role of the curvaton.

5.4 Examples

We are now ready to demonstrate through a few examples how our model can lead to observable statistical anisotropy in the spectrum and bispectrum of the curvature perturbation. In all cases we consider inflation at the scale of grand unification, with $H_* \sim 10^{14}$ GeV, which is favoured by observations [cf. Eq. (5.26)].

Even though there is no compelling reason why the value of the physical vector field cannot be super-Planckian (since $\Omega_A < 1$)⁸, we make the conservative choice $W_* \sim M_P$ in the example below. In view of Eq. (5.16), this severely restricts N_x to small values ($N_x \leq 6$).

5.4.1 Prompt reheating

Consider first the case of prompt reheating, when $\Gamma \sim H_*$. Then, using also Eq. (5.22), we can recast Eq. (5.18) as

$$\frac{e^{-2N_x} H_*^2}{\sqrt{g\zeta^2} M_P} < M < \frac{e^{-2N_x}}{\sqrt{g\zeta^2}} H_* . \quad (5.28)$$

Using that $10 \text{ TeV} < M < H_*$, the above gives

$$\ln \left[(g\zeta^2)^{-1/4} \sqrt{\frac{H_*}{M_P}} \right] \lesssim N_x \lesssim \ln \left[(g\zeta^2)^{-1/4} \sqrt{\frac{H_*}{10 \text{ TeV}}} \right] . \quad (5.29)$$

Employing Eq. (5.3) and considering $H_* \sim 10^{14}$ GeV, we obtain

$$1 \lesssim N_x \lesssim 18 . \quad (5.30)$$

This range is further truncated if we postulate $W_* \lesssim M_P$ as mentioned. Now, with prompt reheating Eq. (5.11) becomes

$$g \sim \zeta^{-2} e^{-4N_x} \left(\frac{W_*}{M_P} \right)^2 , \quad (5.31)$$

which is independent of the value of M . If we choose $W_* \sim M_P$ then $g \sim 0.1$ for $N_x \approx 5.5$. With these values Eq. (5.15) gives

$$\|f_{\text{NL}}^{\text{eq}}\| \sim 0.01 \times \left(\frac{H_*}{M} \right) . \quad (5.32)$$

Thus, we can obtain observable non-Gaussianity ($\|f_{\text{NL}}^{\text{eq}}\| \gtrsim 1$) for $10^{10} \text{ GeV} \leq M \leq 10^{12} \text{ GeV}$.

⁸This is in contrast to scalar fields, where super-Planckian values are expected to blow-up non-renormalisable terms in the scalar potential and render the perturbative approach invalid.

5.4.2 Intermediate reheating scale

In this example we assume $\Gamma \sim M$. Then, following the same process as above, we arrive at

$$\ln \left[(g\zeta^2)^{-1/4} \sqrt{\frac{H_*}{M_P}} \right] \lesssim N_x \lesssim \ln \left[(g\zeta^2)^{-1/4} \left(\frac{H_*}{10 \text{ TeV}} \right)^{1/6} \right]. \quad (5.33)$$

Employing Eq. (5.3) and considering $H_* \sim 10^{14} \text{ GeV}$, we obtain

$$1 \lesssim N_x \lesssim 10. \quad (5.34)$$

This range is further truncated if we postulate $W_* \lesssim M_P$. Because of this, let us choose $N_x \approx 3$. Then, from Eq. (5.18), it is straightforward to obtain

$$10^{-6} H_* < M < H_*. \quad (5.35)$$

With $W_* \sim M_P$, Eq. (5.11) becomes

$$g \sim \zeta^{-2} e^{-4N_x} \left(\frac{M}{H_*} \right)^{4/3}. \quad (5.36)$$

Combining this with Eq. (5.15) we obtain

$$\|f_{\text{NL}}^{\text{eq}}\| \sim \frac{5}{24} \zeta^{-4} e^{-6N_x} \left(\frac{M}{M_P} \right). \quad (5.37)$$

Using $N_x \approx 3$, we find that $\|f_{\text{NL}}^{\text{eq}}\| \sim 100$ can be attained if $M \sim 10^{-7} M_P \sim 10^{11} \text{ GeV}$, which lies comfortably within the range in Eq. (5.35). Using this value, Eq. (5.36) gives $g \sim 0.1$. These values challenge the observational bounds but they can become a little smaller if M is somewhat reduced.

5.4.3 Gravitational reheating

Assume now that the inflaton decays through gravitational couplings. If we further take the inflaton mass to be of order H_* (this is natural in supergravity [31]), then we have

$$\Gamma \sim \frac{H_*^3}{M_P^2}. \quad (5.38)$$

For inflation at the scale of grand unification we find $\Gamma \sim 10^6 \text{ GeV}$.

Now, let us consider that the dilaton rolls throughout inflation, *i.e.* $N_x = 0$. Taking $W_* \sim M_P$, Eq. (5.11) becomes

$$\frac{M}{H_*} \sim (g\zeta^2)^3 \left(\frac{M_P}{H_*} \right)^6, \quad (5.39)$$

where we have used Eq. (5.22) and we have assumed $M > \Gamma$. Employing Eq. (5.3) and considering $H_* \sim 10^{14}$ GeV, the above gives

$$10^{-7}H_* \leq M \leq 10^{-4}H_* . \quad (5.40)$$

Let us take $M \sim 10^{-4}H_* \sim 10^{10}$ GeV $> \Gamma$, which means $\Gamma_A \sim 10^{-6}$ GeV, according to Eq. (5.22). Then, Eq. (5.11) yields $g \sim 0.1$, while Eq. (5.15) gives $\|f_{\text{NL}}^{\text{eq}}\| \sim 10^2$. Again, these values are a bit of a challenge to the observational bounds but they can be relaxed if M is reduced.

From the above examples we see that it is indeed possible to generate observable statistical anisotropy in the spectrum and bispectrum of the curvature perturbation. If we allow for super-Planckian W_* we can increase N_x but no more than $N_x \simeq 20$. This means that the dilaton needs to roll for a substantial number of e-foldings to allow for observable statistical anisotropy. This problem is ameliorated if there is a subsequent period of inflation (*e.g.* thermal inflation, which can contribute about 20 e-foldings or so [32]). In particular, for super-Planckian W_* and for a total period of inflation that lasts $N \gtrsim 60$ e-foldings, of which 20 e-foldings may be generated subsequently by thermal inflation for example, we see that the dilaton must still evolve for about 20 e-foldings. The reheating temperature $T_{\text{reh}} \sim \sqrt{M_P \Gamma}$ in the above examples is rather large and would result in an overproduction of gravitinos, if the latter were stable. This problem is also overcome by adding a late period of inflation since the entropy release can dilute the gravitinos.

6. Moving brane

We now consider the brane whose world volume hosts our vector curvaton to be the D3-brane which is driving inflation. In open string D-brane models of inflation, inflationary trajectories can arise from motion in the radial direction of a warped throat, in which brane motion may be slow or relativistic, leading to slow-roll or DBI inflation respectively. In addition to the radial direction, one may also consider the brane to have non-trivial motion in any of the five angular directions of the throat, giving an inherently multifield scenario. As shown in Ref. [28, 33] for the case of radial motion plus one angular field, motion in the angular directions experiences strong Hubble damping such that the behaviour of the brane very soon tends towards the conventional single field scenario. This situation was recently confirmed in Ref. [34] in which motion in all six directions in the throat is considered. Thus we can assume that for most of the inflationary period, the motion of the brane in the throat is effectively along a single direction. Nevertheless, it is useful to comment on the multifield case since these scenarios overcome the essential problems with single field DBI inflation, which, for example, are related to the lack of consistency of predicted bounds on the scalar-to-tensor ratio [35]. In what follows, we briefly outline the two

possibilities, *i.e.* slow-roll inflation and DBI inflation, where we consider both single field and multifield DBI scenarios. All the results from the previous sections can then just follow straightforwardly.

To take into account the possible effects of relocating our vector curvaton to a moving brane, we consider the same scenario as is discussed in Sec. 3.2 (*i.e.* a canonical vector field modulated by the dilaton field such that $e^{-\phi} \propto a^2$), but with a position field for general brane motion in the radial direction. We therefore rewrite the action in Eq. (2.10) as

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} R - \frac{M_P^2}{4} \partial_\mu \phi \partial^\mu \phi - V(\phi) - h^{-1} [1 + h e^{-\phi} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} + h \partial_\alpha \varphi \partial^\alpha \varphi + 3h^2 e^{-\phi} (\partial_\alpha \varphi \partial^\alpha \varphi \mathcal{F}_{\beta\gamma} \mathcal{F}^{\beta\gamma} - 2\partial_\alpha \varphi \mathcal{F}^{\alpha\beta} \partial^\gamma \varphi \mathcal{F}_{\gamma\beta})]^{1/2} - V(\varphi) + h^{-1} - \frac{m^2}{2} \mathcal{A}_\mu \mathcal{A}^\mu \right\}. \quad (6.1)$$

We consider cosmological scenarios such that all background fields are functions of time only, in which case there is a cancelation of the terms mixed in φ and \mathcal{A}_μ at background level. Nonetheless, in principle this action could still lead to mixed terms in the perturbations, which can be functions of space as well as time. It turns out however, that the mixed terms do not appear in the equations of motion for the perturbations of both φ and \mathcal{A}_μ , as is clear from examining the complete equations of motion which are given below and considering the possible perturbations of the various terms.

The equations of motion for φ and \mathcal{A}_μ calculated from the action in Eq. (6.1) are given respectively by,

$$\begin{aligned} \frac{h'}{h^2} (1 - \sqrt{\Sigma}) + V'(\varphi) + \frac{h' e^{-\phi} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} + \partial_\alpha \varphi \partial^\alpha \varphi + 6h e^{-\phi} [(\partial_\alpha \varphi)^2 \mathcal{F}^2 - 2\partial_\alpha \varphi \mathcal{F}^{\alpha\beta} \partial^\gamma \varphi \mathcal{F}_{\gamma\beta}]}{2\sqrt{\Sigma}} \\ = \frac{\partial_\mu}{2\sqrt{-g}} \left\{ \frac{\sqrt{-g} [2\partial^\mu \varphi + 3h e^{-\phi} (2\mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} \partial^\mu \varphi - 4\partial^\alpha \varphi \mathcal{F}_{\alpha\beta} \mathcal{F}^{\mu\beta})]}{\sqrt{\Sigma}} \right\} \end{aligned} \quad (6.2)$$

$$m^2 \mathcal{A}^\nu = \frac{\partial_\mu}{\sqrt{-g}} \left\{ \frac{\sqrt{-g} (e^{-\phi} \mathcal{F}^{\mu\nu} + 3h e^{-\phi} [2(\partial_\alpha \varphi)^2 \mathcal{F}^{\mu\nu} - 2\partial_\alpha \varphi \mathcal{F}^{\alpha\nu} \partial^\mu \varphi + 2\partial_\alpha \varphi \mathcal{F}^{\alpha\mu} \partial^\nu \varphi])}{\sqrt{\Sigma}} \right\}, \quad (6.3)$$

where

$$\Sigma = 1 + h(\partial_\alpha \varphi)^2 + h e^{-\phi} \mathcal{F}^2 + 3h^2 e^{-\phi} [(\partial_\alpha \varphi)^2 \mathcal{F}^2 - 2\partial_\alpha \varphi \mathcal{F}^{\alpha\beta} \partial^\gamma \varphi \mathcal{F}_{\gamma\beta}].$$

Neglecting the mixed terms at both background and perturbation level, and considering the derivatives acting on \mathcal{A}_μ to be small while keeping those acting on the position field to be general, we may expand $\sqrt{\Sigma}$, and, keeping only up to quadratic

order in \mathcal{F} , the equations of motion then become

$$V'(\varphi) + \frac{h'}{h^2} \left[1 - \sqrt{1 + h(\partial_\alpha \varphi)^2} \right] + \frac{h'}{2h} \frac{(e^{-\phi} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} + \partial_\alpha \varphi \partial^\alpha \varphi)}{\sqrt{1 + h(\partial_\alpha \varphi)^2}} = \frac{\partial_\mu}{\sqrt{-g}} \left[\frac{\sqrt{-g} \partial^\mu \varphi}{\sqrt{1 + h(\partial_\alpha \varphi)^2}} \right], \quad (6.4)$$

for the inflaton field, and

$$m^2 \mathcal{A}^\nu = \frac{\partial_\mu}{\sqrt{-g}} \left[\frac{\sqrt{-g} e^{-\phi} \mathcal{F}^{\mu\nu}}{\sqrt{1 + h(\partial_\alpha \varphi)^2}} \right], \quad (6.5)$$

for the vector field.

An important feature of (6.5) to note is the new form of the gauge kinetic function

$$f = \frac{e^{-\phi}}{\sqrt{1 + h \partial_\alpha \varphi \partial^\alpha \varphi}} = \gamma_\varphi e^{-\phi}. \quad (6.6)$$

Employing the metric (2.1) into these equations, we find that the equations of motion for the background fields $\varphi(t)$ and $\mathcal{A}_\mu(t)$ are given respectively by

$$\ddot{\varphi} - \frac{h'}{h^2} + \frac{3h'}{2h} \dot{\varphi}^2 + 3H \dot{\varphi} \frac{1}{\gamma_\varphi^2} - \frac{h'}{h} e^{-\phi} \left(\frac{\dot{\mathcal{A}}}{\gamma_\varphi a} \right)^2 + \left(V'(\varphi) + \frac{h'}{h^2} \right) \frac{1}{\gamma_\varphi^3} = 0 \quad (6.7)$$

and

$$\mathcal{A}_t = 0, \quad (6.8)$$

$$\ddot{\mathcal{A}} + \dot{\mathcal{A}} \left(H + \frac{\dot{f}}{f} \right) + \frac{m^2}{f} \mathcal{A} = 0. \quad (6.9)$$

In the absence of the vector field, Eq. (6.7) reduces to the standard equation of motion for the DBI inflaton (see Ref. [15]), where we note that now, since the scalar field is homogenised by inflation,

$$\gamma_\varphi = \frac{1}{\sqrt{1 - h \dot{\varphi}^2}}. \quad (6.10)$$

Eq. (6.10) gives the Lorentz factor for brane motion in the internal space, and is a direct generalisation of the Lorentz factor for a relativistic point particle. In an AdS throat, the warp factor is simply given by

$$h = \frac{\lambda}{\varphi^4} \quad (6.11)$$

where $\lambda = g_{YM}^2$ is the 't Hooft coupling, and we require $\lambda \gg 1$ such that the system may be described via the gravity side of the AdS/CFT correspondence. The brane must obey the causal speed limit in the bulk, which is equivalent to requirement that

γ_φ remains real at all times. Given the form of the warp factor in Eq. (6.11), we see that warping becomes significant as $\varphi \rightarrow 0$, therefore at small φ the velocity of the relativistic brane is forced to decrease. Indeed, in Ref. [15] it is shown that at late times

$$\varphi(t) \rightarrow \frac{1}{t}, \quad (6.12)$$

which implies that for a pure AdS throat, the brane takes an infinite time to cross the horizon. A realistic throat may be approximated as AdS in the regions of interest but has a finite cut-off at the IR end, therefore the brane may cross the horizon in finite time. The fact that the brane is forced to slow down as it moves towards the horizon leads to inflationary trajectories in this region.

In an AdS geometry, γ_φ may indeed become arbitrarily large at late times, and this leads to a suppression of all but the first three terms in Eq. (6.7). Therefore, we see that in this case the vector term has a negligible impact on the dynamics of the inflaton, along with the potential term and friction term, as soon as the brane starts to approach the speed limit $\dot{\varphi} = \varphi^2/\sqrt{\lambda}$. The same is true for the potential and friction terms in the case of standard DBI inflation in an AdS throat (see Ref. [15]).

In a Klebanov-Strassler throat, the behaviour of γ_φ is such that its maximum value is reached almost immediately as the brane moves from the UV end of the throat, dropping for subsequent times. At late times, when the brane is moving in the IR region of the throat, the value of γ_φ is roughly constant, remaining within a single order of magnitude. Ultimately $\gamma_\varphi \rightarrow 1$, when the brane stops. This means that the vector, friction and potential terms in Eq. (6.7) are no longer suppressed for later times. In this case the vector \mathcal{A}_μ can have an influence on the dynamics of the inflaton. In the standard DBI scenarios in Klebanov-Strassler throats, *i.e.* without the vector contribution, the presence of non-negligible potential and friction terms in the dynamics of the inflaton does not change the result: inflation still takes place in the throat. For our case, the presence of the vector field may contribute an effective term in the potential for the inflaton, along the lines of what has been demonstrated in Ref. [36]. For the time being we focus on the simpler AdS case, such that the vector term is subdominant in the dynamics of the inflaton, and we can treat the system as undergoing standard DBI inflation in an AdS background. However, further work is currently in progress that will assess the impact of the vector backreaction in a Klebanov-Strassler throat.

Let us now consider conventional slow-roll, which is possible if the potential admits a particularly flat section. When the brane is slowly rolling along a flat section of its potential in the throat, the derivatives of both the vector as well as the position field are small and we can expand the $\sqrt{\Sigma}$ factor in Eqs. (6.2). Keeping only those terms that are up to quadratic order in the derivatives of both of the fields, we recover the standard Klein-Gordon equation for a minimally coupled scalar field,

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0. \quad (6.13)$$

We can now implement the vector curvaton scenario in this set-up as follows. Assuming that the vector field and the dilaton give a subdominant contribution to the energy density during inflation, the energy density and pressure calculated from the action in Eq. (6.1) are given by

$$\rho = \frac{1}{h}(\gamma_\varphi - 1) + V, \quad (6.14)$$

$$p = \frac{1}{h} \left(1 - \frac{1}{\gamma_\varphi} \right) - V. \quad (6.15)$$

We consider the brane to be moving relativistically, therefore γ_φ is large. As discussed above, in the limit of strong warping the velocity of the brane is forced to decrease, hence the energy density becomes dominated by the potential. For large γ_φ and strong warping, the pressure is clearly also dominated by the potential. This illustrates how inflation can arise in a DBI scenario.

Taking into account the new form of the gauge kinetic function, we see that if $\gamma_\varphi \neq 1$ the scaling necessary for statistical anisotropy could in principle be spoilt by new powers of the scale factor that are introduced as a result of the inflaton. As shown in Ref. [15], for DBI inflation in an AdS geometry with a warp factor as in Eq. (6.11), the scale factor $a(t) \rightarrow a_0 t^{1/\epsilon}$ at late times, where ϵ is a generalisation of the slow-roll parameter and is given by

$$\epsilon = \frac{2M_P^2}{\gamma_\varphi} \left(\frac{H'}{H} \right)^2, \quad (6.16)$$

such that $\ddot{a}/a = H^2(1 - \epsilon)$ and one obtains de Sitter expansion for $\epsilon \rightarrow 0$. For a background expanding in this way, the vector field will undergo gravitational particle production as outlined in Sec. 3.2 and obtain a scale invariant spectrum of superhorizon perturbations as long as we still have $f \propto a^2$ and as long as the vector field remains light (the vector mass m does not depend on the inflaton and therefore the condition $m \propto a$ is not impacted, however the physical mass $M = m/\sqrt{f}$ is impacted).

It is further shown in Ref. [15] that $\gamma_\varphi \propto t^2$ at late times, *i.e.* $\gamma_\varphi \propto a^{2\epsilon}$ which means that our gauge kinetic function is now $f = e^{-\phi} \gamma_\varphi \propto a^{2+2\epsilon}$. This could contribute a small degree of scale dependance to the power spectrum of vector perturbations, however clearly the scaling $f \propto a^2$ still holds. Furthermore, as shown in Ref. [9], when $f \propto a^{2(1+\epsilon)}$, the spectral tilt for the transverse and longitudinal components of the vector field are different with the corresponding spectral indexes being

$$n_{L,R} - 1 = -\frac{8}{3}\epsilon \quad \text{and} \quad n_{\parallel} - 1 = 2\epsilon, \quad (6.17)$$

i.e. the transverse spectrum is slightly red and the longitudinal is slightly blue.

The physical mass of the vector field is now given by $M = m/\sqrt{f} \propto 1/\sqrt{\gamma_\varphi} \propto a^{-\epsilon}$, which means that M now experiences a slight evolution during inflation. In particular, when $\gamma_\varphi \gg 1$ the magnitude of the vector mass is decreased such that any evolution of M only serves to make the condition $M \ll H$ easier to fulfill.

In addition to the suppression of the vector term by γ_φ , we saw before that for a physical vector mass $M \ll H$ during inflation and a gauge kinetic function $f \propto a^2$, the equation of motion for \mathcal{A}_μ , given in Eq. (3.11), implies that the vector field freezes at constant amplitude, such that $\dot{\mathcal{A}}_\mu$ (which appears in Eq.(6.7)) is expected to be very small during this time. The same behaviour occurred during inflation for the non-canonical vector field, as can be seen in Figs. 1 and 2. Note also, that the vector field is coupled to the inflaton through the Lorentz factor in Eq. (6.10), which features the derivatives of φ , which are expected to be small during slow-roll inflation.⁹

In Refs. [16, 37] the potential for a D3-brane has been explicitly calculated taking into account all corrections from fluxes and bulk objects, and the results show that it is possible, albeit with fine-tuning, to obtain a flat region in which a slow-roll phase could occur. Similarly, an explicitly calculated potential is studied in Ref. [38], in which it is shown that a sufficiently long period of inflation as well as a correct spectrum of perturbations can be achieved from the combination of a slow-roll and DBI phase, where slow-roll is obtained by fine-tuning the potential in the region close to the tip. In such pictures in which the dominant contribution to the curvature perturbation can be successfully generated by the inflaton, the modulated vector curvaton considered in the present work will add the new feature of measurable statistical anisotropy.

Let us now comment on the multifield case. In the simplest case one may consider a generalization to a two field model in which, in addition to its motion in the radial direction, the brane moves in one of the five angular directions of the warped throat. Such a scenario is considered in Ref. [28, 33] and we briefly discuss this picture here to illustrate the multifield generalisation of our work, keeping in mind that the dominant behaviour of the brane is always well approximated by a single field scenario at the times of interest to us.

In general for a multifield scenario, several of the terms contained in the determinant in Eq. (2.6) computed for the DBI action in Eq. (2.4) may no longer vanish after the antisymmetrisation. However, these terms become subdominant as soon as the brane tries to move radially only, and therefore we do not need to consider them to explain the essentials of the multifield picture. The important point is that the vector field and the scalar fields are decoupled for late times at both background and perturbation level, as we saw for the single field case. Considering only the dominant

⁹This is in contrast to Ref. [36], where the kinetic function of the vector field is modulated by the inflaton field itself and not by its derivatives.

terms in Eq. (2.6), the transition to a multifield scenario will impact the form of γ_φ , which now becomes,

$$\gamma_\varphi = \frac{1}{\sqrt{1 - h \dot{\varphi}^i \dot{\varphi}^j g_{ij}}}, \quad (6.18)$$

where g_{ij} is the metric on the internal space.

The energy density and pressure calculated from this action are analogous to Eqs. (6.14) and (6.15), but where γ_φ is now given by (6.18). Once the brane starts to settle onto a radial trajectory, we recover single field DBI inflation and the familiar form of γ_φ given in (6.10). All of the results outlined before for the single field case are then applicable here. The upshot is that we now have a scenario in which the dominant contribution to the curvature perturbation is generated consistently by the multifield DBI inflaton, and the vector curvaton introduced in the present work can induce a measurable level of statistical anisotropy, providing a concrete mechanism for the generation of such a feature whose existence is implied by precision measurements of the CMB.

7. Conclusions

In this paper we have explored the first realisation of a string vector curvaton scenario, where the vector field which lives on a D3-brane plays the role of the vector curvaton. We have investigated how this scenario can affect the observed curvature perturbation ζ in the Universe. We have first considered the case in which the vector curvaton brane is stationary and inflation occurs in some other sector, for example via warped $D\bar{D}$ inflation, or via the motion of a different D3-brane. We have demonstrated that, for suitable values of the parameters, such a vector curvaton can generate observable statistical anisotropy in the spectrum and bispectrum of ζ provided that the dilaton field, which is a spectator field during inflation, varies with the scale factor as $e^\phi \propto a^{-2}$ when the cosmological scales exit the horizon. If this is the case, we have shown that both the transverse and the longitudinal components of the vector field obtain a scale-invariant superhorizon spectrum of perturbations. However, particle production is anisotropic, which means that the vector curvaton cannot generate ζ by itself but it can give rise to observable statistical anisotropy. In view of the forthcoming data from the Planck satellite, this is a finding that will be testable in the very near future, which renders our model both timely and directly falsifiable from the observations. Indeed, in our model, statistical anisotropy in the bispectrum is predominant. This means that non-Gaussianity has to have a strong angular modulation on the microwave sky, which may or may not be found by Planck. However, our model can still produce observable statistical anisotropy in the spectrum even if its contribution to the bispectrum is negligible (and vice-versa, see also Ref. [39]).

We then demonstrated that these results are robust when we allow for the possibility that the same brane which hosts the vector curvaton, is also responsible for driving cosmological inflation, where inflation can be of either the slow roll or the DBI variety. All the results obtained for the stationary brane case follow and we again can obtain measurable statistical anisotropy both in the spectrum and the bispectrum. Moreover, in the case of DBI inflation, the constraints on the vector mass can be considerably improved. Furthermore, since DBI inflation also contributes to the generation of large non-Gaussianities of the equilateral type, in this case we have two different sources for large Gaussian deviations. Thus we have shown that the presence of several light fields in string theory models of cosmology can provide us with a unique source for distinctive features, which can help us to distinguish such models from pure field theory models. Indeed, cosmology may prove to be the best arena in which to test the predictions of string theory, and the search for such features as have here been proposed makes progress in this direction. Certainly these possibilities deserve further investigation, and in the present work we have only begun to explore the prospects for sources of stringy statistical anisotropies in the power spectra.

A comment is in order here. As we have shown, our results are strongly based on the assumption that the dilaton rolls for a considerable amount of time, and in particular, for at least 20 e-folds. Based on the results of [16], we can expect that a large displacement of the dilaton away from its minimum does not perturb the inflaton potential, as long as we ensure that at all times we remain with the regime in which $g_s < 1$. However, we still require that the minimum of the dilaton potential is located at an extremely weak coupling, $g_s \ll 1$. We have commented on a possible way to ameliorate this problem via successive periods of inflation. However, this remains a problem which certainly needs to be addressed in more detail.

It should also be pointed out that in the present work we did not explore the new terms that arose in our equations of motion for the perturbations of the non-standard vector field, discussed in Sec. 3.1. Note that in this case, the results of the vector curvaton paradigm introduced in Ref. [4, 8] do not apply in principle. The interpretation of these new terms, in particular the question regarding whether or not they may lead to interesting features in a cosmological context, is left for future work.

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A. Mass generation mechanism for $U(1)$ field

Consider a Lagrangian of the following form [40]:

$$\mathcal{L} = -\frac{e^\phi}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{e^{-\phi}}{4\tilde{g}^2} F^{\mu\nu} F_{\mu\nu} + \frac{c}{4} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma}, \quad (\text{A.1})$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$, \tilde{g} and c are arbitrary constants and we have included a dilaton-like coupling to the fields. The Lagrangian (A.1) describes a massless two-form field $B_{\mu\nu}$, with one degree of freedom, coupling to a massless gauge field A_μ , with 2 degrees of freedom, plus the dilaton field. The fact that the two-form field has one degree of freedom is evident by the fact that in four dimensions, it transforms under the little group $SO(2)$. We will see that in its dual form, this Lagrangian describes a gauge field A_μ with 3 degrees of freedom, *i.e.* the single degree of freedom carried by the two-form field is “eaten” by the gauge field to provide a mass, and the two-form field no longer appears.

To arrive at the dual form, we make an intermediate step which involves rewriting Eq. (A.1) by integrating the coupling term by parts, and then imposing the constraint $H = dB$ by way of a Lagrange multiplier field η . Integration by parts changes the form of the coupling term from $\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma}$ to $\epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} A_\sigma$, and so eliminates $B_{\mu\nu}$ from the Lagrangian. To retain the same information as was present in the original form, we need to impose the constraint $H = dB$, however, as we have already eliminated $B_{\mu\nu}$, we formulate the constraint in terms of the new field $H_{\mu\nu\rho}$ as $dH = 0$ (which is of course true in the case that $H = dB$). The Lagrangian in Eq. (A.1) can be thus rewritten as:

$$\mathcal{L} = -\frac{e^\phi}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{e^{-\phi}}{4\tilde{g}^2} F^{\mu\nu} F_{\mu\nu} - \frac{c}{6} \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} A_\sigma - \frac{c}{6} \eta \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma}. \quad (\text{A.2})$$

Now, integrating by parts the last term in Eq. (A.2),

$$\mathcal{L} = -\frac{e^\phi}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{e^{-\phi}}{4\tilde{g}^2} F_{\mu\nu} F^{\mu\nu} - \frac{c}{6} \epsilon^{\mu\nu\lambda\beta} H_{\mu\nu\lambda} (A_\beta + \partial_\beta \eta) \quad (\text{A.3})$$

and solving for H , we find:

$$H^{\mu\nu\lambda} = -c e^{-\phi} \epsilon^{\mu\nu\lambda\beta} (A_\beta + \partial_\beta \eta).$$

Inserting this back into Eq. (A.3), we find

$$\mathcal{L} = -\frac{e^{-\phi}}{4\tilde{g}^2} F^{\mu\nu} F_{\mu\nu} - \frac{c^2 e^{-\phi}}{2} (A_\sigma + \partial_\sigma \eta)^2. \quad (\text{A.4})$$

Normalising the kinetic term, we see that the gauge field A_μ has acquired a mass $m^2 = \tilde{g}^2 c^2 e^{-\phi}$. Notice the the scalar η can be gauged away via a gauge transformation of $A \rightarrow A + \partial\Lambda$, thus we are left with only the mass term. By absorbing the scalar field η into the gauge field, we have explicitly chosen a gauge.

In our set-up discussed in the main text, this mechanism is realised via the coupling of the gauge field F_2 to the RR two form C_2 (see Eq. (2.7)). The kinetic term for C_2 descends from the 4D components of the RR field strength, $F_3 = dC_2$. This arises from the ten dimensional type IIB action. The relevant piece in the Einstein frame, is given by

$$\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_{10}} \left(-\frac{e^\phi}{12} F_{\mu\nu\lambda} F^{\mu\nu\lambda} \right), \quad (\text{A.5})$$

where $2\kappa_{10}^2 = (2\pi)^7 (\alpha')^4$ [17]. After dimensional reduction to four dimensions this becomes:

$$\frac{M_P^2}{2} \int d^4x \sqrt{-g_{10}} \left(-\frac{e^\phi}{12} F_{\mu\nu\lambda} F^{\mu\nu\lambda} \right), \quad (\text{A.6})$$

where we have used that

$$\frac{1}{2\kappa_{10}^2} \int d^6x h \sqrt{-g_6} = \frac{V_6}{2\kappa_{10}^2} = \frac{M_P^2}{2}. \quad (\text{A.7})$$

where h is the warped factor. Using this and (2.7), we find the appropriate Lagrangian for the mass generation in our set-up,

$$\mathcal{L}_{\text{mass}} = -\frac{e^\phi}{12} \left(\frac{M_P^2}{2} \right) F^{\mu\nu\rho} F_{\mu\nu\rho} - \frac{T_3 (2\pi\alpha')^2 e^{-\phi}}{4} F^{\mu\nu} F_{\mu\nu} + \frac{T_3 (2\pi\alpha')}{4} \epsilon^{\mu\nu\rho\sigma} \mathcal{C}_{\mu\nu} F_{\rho\sigma}. \quad (\text{A.8})$$

Rescaling the 2-form as, $C_2 = \frac{\sqrt{2}}{M_P} \tilde{C}_2$, the Lagrangian takes the form of Eq. (A.1)¹⁰

$$\mathcal{L}_{\text{mass}} = -\frac{e^\phi}{12} \tilde{F}^{\mu\nu\rho} \tilde{F}_{\mu\nu\rho} - \frac{e^{-\phi}}{4\tilde{g}^2} F^{\mu\nu} F_{\mu\nu} + \frac{c}{4} \epsilon^{\mu\nu\rho\sigma} \tilde{C}_{\mu\nu} F_{\rho\sigma}, \quad (\text{A.9})$$

where

$$\tilde{g}^2 = \frac{1}{T_3 (2\pi\alpha')^2}, \quad c = \frac{T_3 (2\pi\alpha') \sqrt{2}}{M_P}. \quad (\text{A.10})$$

Thus the dilaton dependent mass for the vector field is given by

$$m^2 = \tilde{g}^2 c^2 e^{-\phi} = \frac{2 T_3 e^{-\phi}}{M_P^2} = \frac{(2\pi)^4 M_s^2 e^{-\phi}}{\mathcal{V}_6}, \quad (\text{A.11})$$

where $M_s = \alpha'^{-1/2}$ is the string scale, $\mathcal{V}_6 = V_6/\ell_s^6$ is the dimensionless six dimensional volume and we have used that

$$T_3 = (2\pi)^{-3} (\alpha')^{-2}, \quad M_P^2 = 2(2\pi)^{-7} \mathcal{V}_6 M_s^2. \quad (\text{A.12})$$

¹⁰Note that in our case, $C_2 = \mathcal{C}_2$.

Going back to the WZ action for the D3-brane in Eq. (2.7), using $C_4 = \sqrt{-g} h^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$ and the mass term discussed above, we have

$$S_{\text{WZ}} = q \int d^4x \sqrt{-g} \left(h^{-1} - \frac{m^2}{2} \mathcal{A}_\mu \mathcal{A}^\mu + \frac{\mathcal{C}_0}{8} \epsilon^{\mu\nu\lambda\beta} \mathcal{F}_{\mu\nu} \mathcal{F}_{\lambda\beta} \right), \quad (\text{A.13})$$

where here $\tilde{g}\mathcal{A}_\mu = A_\mu$ is the canonical normalised gauge field (and correspondingly $\mathcal{F}_{\mu\nu}$ its field strength). Further, $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita tensor, such that $\epsilon_{0123} = \sqrt{-g}$.

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